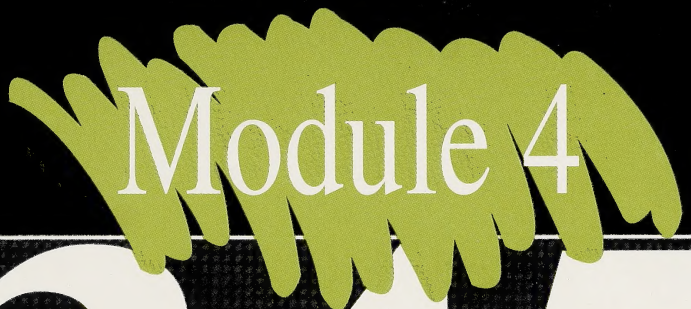
MATHEMATICS
Preparation 10

Polynomials



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MATHEMATICS
Preparation 10

Module 4
Polynomials

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Mathematics Preparation 10
Module 4: Polynomials
Student Module Booklet
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Teachers	✓
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Other	

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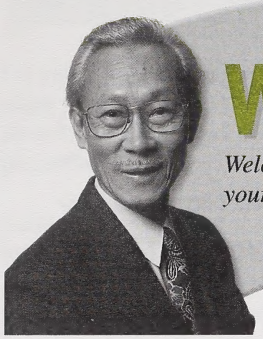

Welcome

Welcome to Module 4. We hope you enjoy your study of "Polynomials."

Course Overview

Mathematics Preparation 10 consists of five modules and a final test. Each module is worth 1 credit. Discuss with your teacher how many modules you need to complete to prepare yourself for Pure Mathematics 10 or Applied Mathematics 10.

Mathematics Preparation 10

Module 1 Exploration of Numbers	Module 2 Number Connections
Module 3 Patterns and Algebra	
Module 4 Polynomials	Module 5 Shape and Space

The document you are now reading is called a Student Module Booklet. It has accompanying Assignment Booklets.

You will find many visual cues or icons throughout this Student Module Booklet. Read the following statements to discover what each icon prompts you to do.

Use the Internet to explore a topic.

Use a scientific calculator.

Use the suggested answers in the Appendix to correct activities.

Answer the questions in the Assignment Booklet.

Throughout the course, you will be given instruction and practice on each of the following mathematical processes:

- connecting mathematical ideas
- developing and using estimation strategies
- developing and using mental math strategies
- using technology appropriately
- developing and using problem-solving strategies
- reasoning and justifying your answers
- using visualization to assist in processing information
- solving problems
- communicating mathematically

For example, to help you develop your problem-solving skills, several problem-solving strategies are explained in the Appendix of each Student Module Booklet, and you will be given several non-routine problems to solve in each module. Watch for the heading "Now Try This."

To help you develop your communication skills in mathematics, you will be asked to do several journal writings in each module. Watch for the heading "Looking Back."

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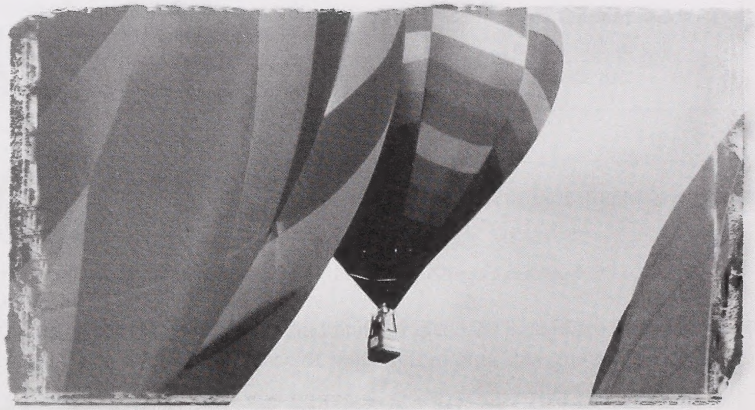


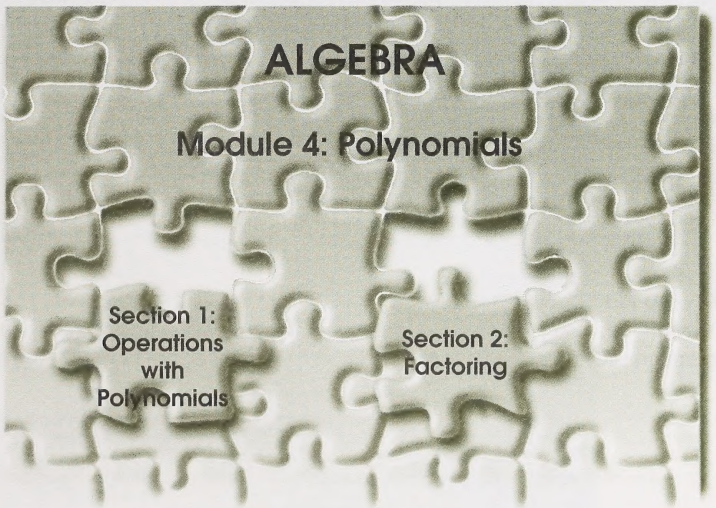
NOVA DEVELOPMENT CORPORATION

Module Overview

Do you like piecing jigsaw puzzles together? A large jigsaw puzzle contains many interconnecting pieces and can be difficult to put together. But if you pay attention to the various patterns and colours of the picture, you can find the pieces that fit together.

Similarly, the study of algebra is composed of many interconnecting parts. You have completed some of the pieces of the algebra puzzle previously.

In Module 4, you will study more about algebra as you examine polynomials. You will review what polynomials are, their use, and how to work with them. You will model polynomials using algebra tiles; add, subtract, multiply, and divide various types of polynomials; and use a variety of techniques to factor and simplify polynomials.

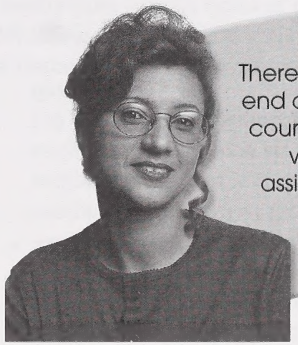


Assessment

Your mark for this module will be determined by how well you complete the assignments at the end of each section. In this module, you must complete three assignments. The mark distribution is as follows:

Assignment Booklet 4A	
Section 1 Assignment	**40 marks**
Assignment Booklet 4B	
Section 2 Assignment	**20 marks**
Final Module Assignment	**40 marks**
TOTAL	**100 marks**

When doing the assignments, work slowly and carefully. You must do each assignment independently; but if you are having difficulties, you may review the appropriate section in this Student Module Booklet.



There is a supervised final test at the end of this course. Your mark for the course will be determined by how well you do on the module assignments and the supervised final test.

Stragegies for Completing the Module Successfully

To achieve success in this module, be sure to work slowly and systematically through the Student Module Booklet. Remember, your work in the Student Module Booklet will prepare you for your module assignments and final test.

Following are some strategies for completing the module successfully:

- Try to set realistic goals for each day. Once you've set these goals, stick to them.
- Read all the instructions in each Student Module Booklet carefully; answer all the questions in your mathematics binder; and check your answers by comparing your responses with the suggested answers in the Appendix.
- Keep a section of your mathematics binder for your journal entries. Get in the habit of describing new concepts, procedures, and strategies in your own words. Record useful ways to help you remember what a concept means. Make graphic organizers (such as context webs, Venn diagrams, and charts) to help you connect mathematical ideas.
- Ask someone who is taking Mathematics Preparation 10 to be your study partner. You will find that having a friend to discuss mathematics with will make your studying more enjoyable.
- If you need assistance from your teacher or your study partner, you may find it helpful to write your questions in the journal section of your mathematics binder before speaking to your teacher or study partner. This will help you pinpoint your problem.
- Take care when completing your module assignments. Be sure you have completed each part of the Assignment Booklets, and proofread the assignments before you submit them. Remember to review your module assignments after they are corrected; then file them in your binder so you can review your assignments and mathematics notes before you write the final test.

Good luck!

SECTION 1

Operations with Polynomials

A good house can only be built on a properly constructed foundation. If the foundation is not laid properly, cracks will soon appear in the walls, doors and windows will not fit properly, and numerous other defects will result.

Similarly, you cannot build a good knowledge of algebra, particularly polynomials, without a firm grasp of the basics. You must lay a solid foundation of basic ideas concerning polynomials before you can carry on with operations involving polynomials.

The polynomial foundation will be laid in the first activity of this section. There you will interpret and classify polynomials. For the remainder of this section, you will build the polynomial framework by adding and subtracting polynomials, becoming familiar with multiplying various types of polynomials, and dividing polynomials by monomials. Becoming confident in working with polynomials will provide a sturdy framework for further explorations with polynomials in Section 2.

Activity 1: Interpreting Polynomials

When driving a vehicle, the driver should know the meaning of every road sign he or she encounters. In Alberta, when people apply for their Class 5 driver's licence (the basic driver's licence), they must go through a vision screening, supply a medical report, and take a written test and a road test. One part of the written test requires the applicant to know the meaning of various road signs, like the ones given on the right. Do you know the meaning of these road signs?

Similarly, before proceeding in your work with algebra, you must become familiar with what polynomials represent.

In this activity, you will identify the various terms involved with polynomials, and you will classify polynomials.

A polynomial is an algebraic expression in which there are no variables in a denominator, no variables under a root sign, and no variables with fractional exponents.

A polynomial may have one or more terms.

A term is a mathematical expression consisting of the product of only numbers and variables with whole-number exponents. The number factor is called the numerical coefficient, and the variable factor (or factors) is called the literal coefficient. A term that consists of a number by itself is called a constant. Terms in a polynomial are separated by either a plus or minus sign.

Consider $4n^2$, the first term of the polynomial $4n^2 + mn - 8$.

- The numerical coefficient is 4.
- The literal coefficient is n^2.

Consider mn, the second term of the polynomial $4n^2 + mn - 8$.

- The numerical coefficient is understood to be 1.
- The literal coefficient is mn.

When a term has more than one variable factor, the variable factors are arranged in alphabetical order.

Consider -8, the third term of the polynomial $4n^2 + mn - 8$.

- The numerical coefficient is -8.
- There is no literal coefficient for a constant term.

Terms with identical literal coefficients are called like terms. Both the variables and exponents must be the same. Examples of like terms are $3a$ and $2a$, n^2 and $-4n^2$, and a^2b and $-3a^2b$.

1. Is each of the following algebraic expressions a polynomial? If not, why not?

a. $4b$ **b.** $4a-\sqrt{2}$ **c.** $\sqrt{a+1}$

d. $\dfrac{a^2-2a+1}{2}$ **e.** $\dfrac{1}{a}-\dfrac{1}{b}$ **f.** $\dfrac{2}{x+1}$

2. Identify the numerical coefficients, literal coefficients, and variables for each of the following terms.

a. $7ab$ **b.** $-8y^3$ **c.** $4xyz$

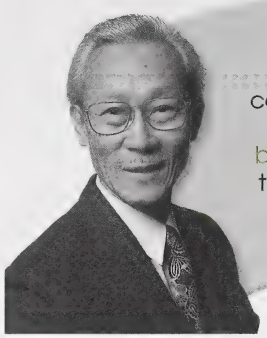
d. $-6z$ **e.** $2x^2y^3$ **f.** $\dfrac{ab}{2}$

3. Group the like terms in the following list.

- m^2n
- $4a$
- $-5ab$
- $2m^2n$
- $-3a^2b$
- $-12a^2b$
- $-5a$
- 6
- $-8m^2n$
- $2mn$
- $-2ab$
- mn
- -9
- $\dfrac{m^2n}{2}$
- $\dfrac{16a}{3}$

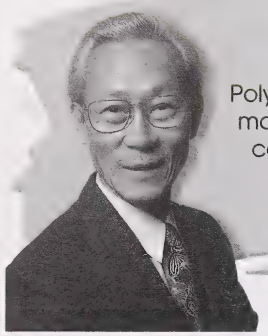
Check your answers by turning to the Appendix, page 93.

There are special names for polynomials that have one, two, or three terms.



A polynomial with one term is called a monomial; a polynomial with two terms is called a binomial; and a polynomial with three terms is called a trinomial.

That must be because "mono" means one, "bi" means two, and "tri" means three.



Polynomials with four or more terms are simply called polynomials.

Ahhh! They ran out of names, did they?

Example

Which of the following are monomials? Which are binomials? Which are trinomials?

- $2a+3b+4c+5d$
- $4ab+c$
- $3abc$
- $7m^2-6m+1$
- $x+y+z$
- $-4y$
- $4c+2d-e$
- $4+3a$
- 14
- $6a^{-2}+\frac{4}{a}-2$
- $8m^2-6$

Solution

The monomials are $3abc$, $-4y$, and 14.

The binomials are $4ab+c$, $4+3a$, and $8m^2-6$.

The trinomials are $7m^2-6m+1$, $4c+2d-e$, and $x+y+z$.

4. Why are two of the expressions in the preceding example not included in the solution?

5. Indicate whether each of these expressions is a monomial, binomial, or trinomial.

a. -7

b. $4xyz$

c. $g+h$

d. $4.5a-b+c$

e. $x^2+x^2y+x^2y^2$

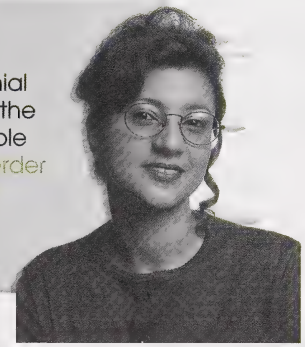
f. $24z-13y-3x$

g. $8-2a$

h. $1.3ab^2c^3d^4$

i. $\frac{a}{2}+\frac{b}{3}+\frac{c}{4}$

Check your answers by turning to the Appendix, page 94.



The terms of a polynomial are usually arranged so the exponents of the variable are either in ascending order or descending order.

Ascending order is from least to greatest. Descending order is from greatest to least.

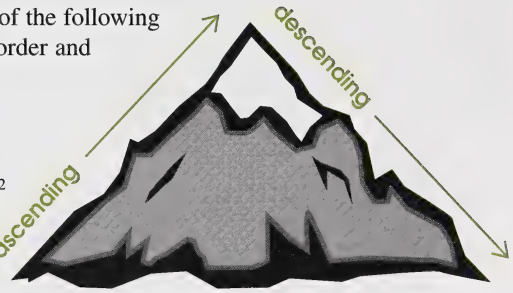
Example

Arrange the terms in each of the following polynomials in ascending order and then in descending order.

a. $5b+b^2+3$

b. $8+3c+5c^3-2c^2$

c. $4-2a^2b+3ab$



Solution

a. Ascending order

$$5b+b^2+3=3+5b+5b^2$$

Descending order

$$5b+b^2+3=b^2+5b+3$$

b. Ascending order

$$8+3c+5c^3-2c^2=8+3c-2c^2+5c^3$$

Descending order

$$8+3c+5c^3-2c^2=5c^3-2c^2+3c+8$$

c. Ascending order

$4-2a^2b+3ab=4+3ab-2a^2b$

Descending order

$4-2a^2b+3ab=-2a^2b+3ab+4$

Note: If there is more than one variable in a polynomial, write the terms in either ascending or descending order of one of the variables.

6. Write each polynomial in ascending order and descending order.

a. $7a+8+3a^2$

b. $2b^3-4b+6+8b^2$

c. $16+2c$

d. $4m^3n^2-2mn+6m^2n-4$

e. $9+3x^3y^2-x^2y^2+3xy$

Check your answers by turning to the Appendix, page 94.

The degree of a term is the sum of the exponents of all of the variables in the term. The degree of a constant term is 0.

The degree of a polynomial is the degree of the term that has the largest sum of exponents.

Example

Copy and complete the following table.

Expression	Degree of Each Term	Degree of Polynomial
$8n^3$		
$4a^2b^3c^4$		
$7a^2+7b^3$		
x^4+7x^3-8		
$2x^2y^4+3x^2y^3-6xy^5$		
$3m^3n^2-8mn^6+7m^3n^3$		

Solution

Expression	Degree of Each Term	Degree of Polynomial
$8n^3$	3	3
$4a^2b^3c^4$	9	9
$7a^2+7b^3$	2, 3	3
x^4+7x^3-8	4, 3, 0	4
$2x^2y^4+3x^2y^3-6xy^5$	6, 5, 6	6
$3m^3n^2-8mn^6+7m^3n^3$	5, 7, 6	7

7. State the degree of each term.

a. $6a^3$ **b.** $-7ab^2$ **c.** abc

d. -12 **e.** $8a^2b^3c$ **f.** $1.4xy^2z$

8. State the degree of each polynomial.

a. $7a+2b^2$

b. x^2+x-1

c. $-2m^3n+4m^2n-2m$

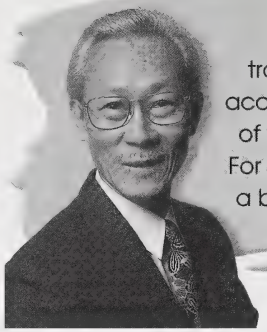
d. $7y+2$

e. $4x^2y+6x^2y^3-14xy+2x^3y^2$

f. $6-2n^3+4n+5n^2$

Check your answers by turning to the Appendix, page 94.

Classifying



Polynomials are traditionally classified according to the number of terms or the degree. For example, $2x^2-7x$ is a binomial of degree 2.

Example

Classify the polynomial $4x^2+2x-6$.

Solution

The polynomial has three terms and is of degree 2.

Therefore, $4x^2+2x-6$ is a second-degree trinomial.

Example

Classify the polynomial $5x^3y^2 - 6xy^2$.

Solution

The polynomial has two terms and is of degree 5.

Therefore, $5x^3y^2 - 6xy^2$ is a binomial of degree 5.

9. Complete the following chart.

Polynomial	Number of Terms (by name)	Degree
$8a^2 + 6a - 2$		
$7b^3 - 3b$		
16		
$2d$		
$2x^2y^3 - 4xy + 7y$		
$10m^2n^4 + 6mn$		
$4y^4 + 3y^3 - 2y^2 + y - 1$		

10. Write a polynomial that has the described characteristics.

a. a monomial of degree 4

b. a second-degree trinomial

c. a first-degree binomial

d. a polynomial of degree 5

e. a trinomial of degree 6

Check your answers by turning to the Appendix, page 95.

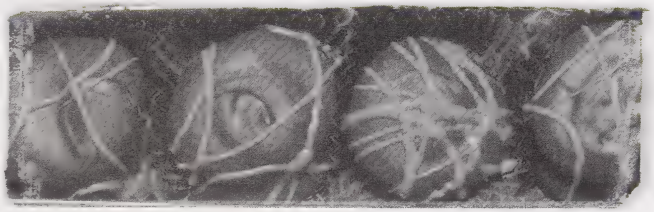
Now Try This

Use one of the problem-solving strategies to solve the following problem.

11. One day, Mr. Wilkinson had a craving for chocolate; so, he bought a box of chocolates on the way home from work. Before he knew it, he had eaten one-sixth of them. He was feeling a bit guilty when he got home, so he offered the rest to his wife, Irene. She thanked him and, while watching television that evening, ate one-fifth of what was left. Their oldest son, Gary, saw the open box when he got home and ate one-quarter of what was left. Janine, their daughter, nibbled away at one-third of those left when she got up the next day. Her younger brother, Anthony, caught her licking her fingers and promptly ate one-half of the remaining chocolates. By the time Mrs. Wilkinson got back to the box of chocolates, there were only four left. How many chocolates were originally in the box?

Solve this problem in two different ways.

Check your answer by turning to the Appendix, page 95.



Looking Back

In this activity, you reviewed the concept of a polynomial.

12. In your journal, complete each sentence using the words from the following list:

degree	expressions
check	binomial
variable	monomial
trinomial	constant

The _____ of the polynomial $5x^2 + 3x - 5$ is two.

An example of a _____ is $2x^3 + 6$.

An example of a _____ is $4x^2y^3$.

An example of a _____ is $4x^2y + 6x + 2y$.

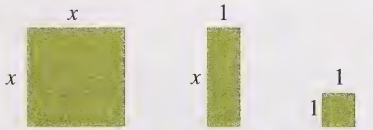
Check your answers by turning to the Appendix, page 97.

Activity 2: Modelling Polynomials

Models are used to show off new clothing designs to prospective buyers. They help give buyers a better picture of what the clothing looks like on a person.

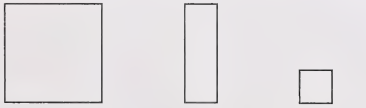
Similarly, using algebra tiles is helpful in visualizing what a polynomial represents.

Each algebra tile has a different name based on its area.



x^2-tile x-tile 1-tile

Each white algebra tile represents the opposite of the tile with the same shape.



$-x^2$-tile $-x$-tile -1-tile

At the back of the Appendix, find the pages with algebra tiles and cut out the tiles. If you are photocopying these pages, you may wish to glue them to heavier paper or boxboard before cutting them out. This will make the tiles easier to handle.

Use the preceding information to help answer questions 1 to 4.

1. **a.** What is the relationship between the length of an x-tile and the length of a side of an x^2-tile.

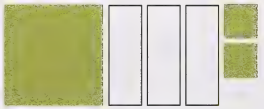
b. What is the relationship between the length of a side of a 1-tile and the width of an x-tile?

2. Write the expression represented by each of these groups of tiles. **Note:** You can use any variable to write the expression.

a.

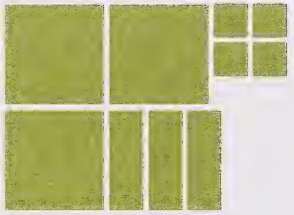
b.

c.



d.

e.



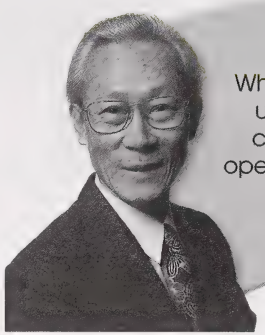
3. Use algebra tiles to model each of the following expressions.

a. $2a^2 - 3a + 1$ **b.** $b^2 + b - 6$ **c.** $-3c^2 + 4c + 2$

d. $d^2 - 2d + 5$ **e.** $e^2 + 3e - 4$

Check your answers by turning to the Appendix, page 97.

Representing Zero



When you modelled integers, you used the concept of zero. The concept of zero is important in operations with polynomials as well.

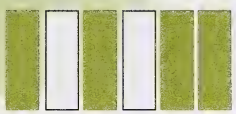
Zero can be represented by using the same number of positive and negative (or opposite) like tiles. You can refer to this as the zero principle.

The following examples show various combinations of tiles including some zero groups.

Example

What do each of the following sets of tiles represent?

a.



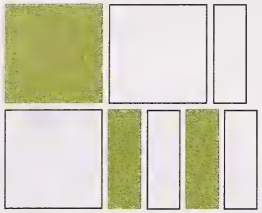
b.

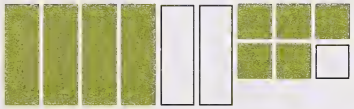
Solution

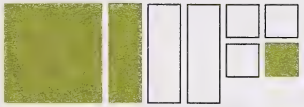
a. The tiles represent $2x$.

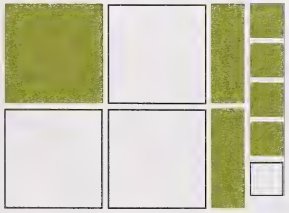
b. The tiles respresent x.

4. What does each group of tiles represent? **Note:** You can use any variable to write the expression.

a. 

b. 

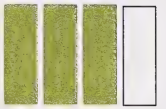
c. 

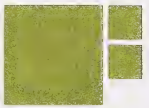
d. 

5. Use algebra tiles to represent zero in three different ways.

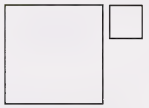
6. What tiles can you add to each group to make the entire group zero?

a.

b. 

c. 

d.

e. 

f.

7. Use algebra tiles to represent each expression; then simplify by collecting like shapes and eliminating the ones that are equivalent to zero.

a. $4a^2 + 2a + 3 - 2a^2 - 3a + 1$

b. $3b^2 + 5 + b^2 + 3b - 7$

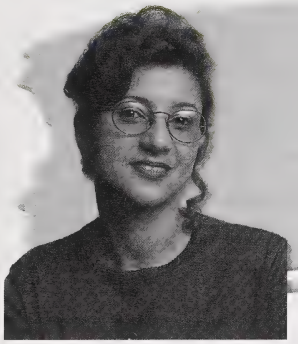
c. $2c^2 - 4c + 1 - 5c^2 + c - 4$

d. $d^2 - d + 7 + 2d^2 + 3d - 2$

8. When collecting the like shapes in question 7, what do you do to simplify?

Check your answers by turning to the Appendix, page 98.

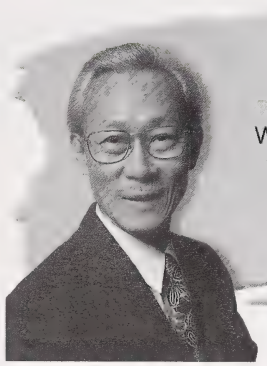
Like Terms



To simplify algebraic expressions, combine like terms.

$-3x^2$ and $5x^2$ are like terms. As a single term, they are equal to $2x^2$.

Like terms are combined or made into a single term by adding their numerical coefficients.



What is an example of like terms?

Example

Simplify the expression $7y^2+8y-4+2y^2-6y+3$ by collecting like terms.

Solution

$$7y^2+8y-4+2y^2-6y+3=\underbrace{7y^2+2y^2}+\underbrace{8y-6y}\underbrace{-4+3}$$

$$=9y^2+2y-1$$

9. Simplify each of the following **without** using algebra tiles. **Note:** It is easier to identify like terms if you read the expression aloud.

a. $7m+2m-3m$

b. $8n^2-2n^2+3n^2$

c. $8y+7+4y-5$

d. $3x^2+2x+4x^2-5x+1$

e. $b^2-2b+4-3b^2+3b-1$

f. $2m+m-m-5$

Note: m means $1m$ and $-m$ means $-1m$.

g. $3y^2+4y-2y^2+6y-1$

h. $x^2-y^2+4x+3y-8+x^2-y^2+2x+3$

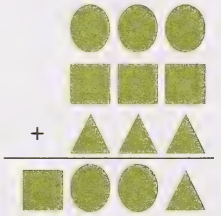
i. $4a^2b+3a^2+7-2b^2+a^2b+4b^2-3$

Check your answers by turning to the Appendix, page 101.

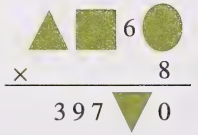
Now Try This

Use one of the problem-solving strategies to solve the following problem.

10. a. Each shape in the sum given represents a certain digit. Determine the missing digits that will make the sum true.



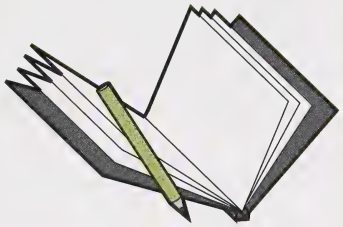
b. Each shape in the given product represents a certain digit. Determine the missing digits that will make the product true.



Check your answers by turning to the Appendix, page 101.

Looking Back

In this activity, you modelled polynomials using algebra tiles. You also reviewed how to simplify algebraic expressions.



11. **a.** To represent x^2, you used a square tile with dimensions x by x. What shape could you use to model x^3? Sketch the shape in your journal.

b. In your journal, sketch the shapes you would use to model $x^3 + 2x^2 - x + 1$.

12. A student was asked to simplify the expression $3x^2 + 5x - 2x + 1$. Here is the student's work:

$$3x^2 + 5x - 2x + 1 = 6x + 1$$

In your journal, explain why $3x^2 + 5x - 2x + 1 \neq 6x + 1$. Use sketches of algebra tiles in your explanation.

Check your answers by turning to the Appendix, page 101.

Activity 3: Adding and Subtracting Polynomials

In Module 1, you used counters to help you understand the concept of adding and subtracting integers. In this activity, you will use algebra tiles to help you understand the concept of adding and subtracting polynomials.

Example

Add the following polynomials.

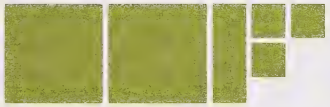
$$\left(2x^2+x+3\right)+\left(-x^2+3x-5\right)$$

Solution

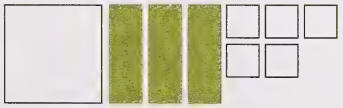
Method 1: Using Models

Model each polynomial.

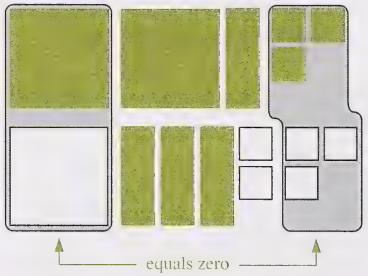
$2x^2+x+3$



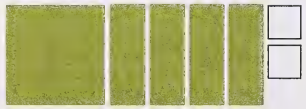
$-x^2+3x-5$



Combine the two sets of tiles, and simplify by removing the zero pairs.



This is the result.



$$\therefore \left(2x^2+x+3\right)+\left(-x^2+3x-5\right)=x^2+4x-2$$

Method 2: Using Paper and Pencil

$$\begin{aligned}\left(2x^2+x+3\right)+\left(-x^2+3x-5\right) &= 2x^2+x+3-x^2+3x-5\\ &= \underbrace{2x^2-x^2}+\underbrace{x+3x}+\underbrace{3-5}\\ &= x^2+4x-2\end{aligned}$$

Example

Add the following polynomials.

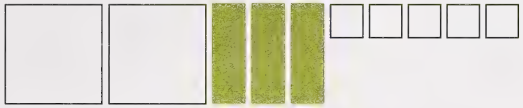
$$\begin{array}{r} -2x^2+3x-5 \\ +\left(4x^2+8\right) \\ \hline \end{array}$$

Solution

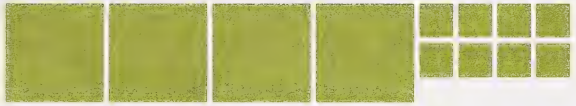
Method 1: Using Models

Model each polynomial.

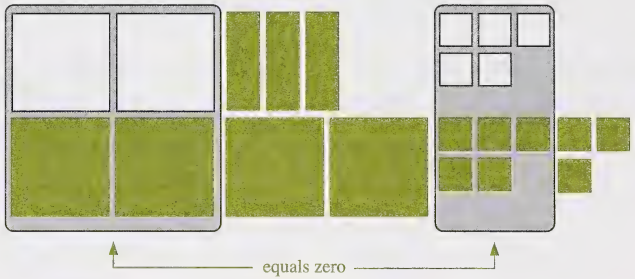
$-2x^2+3x-5$



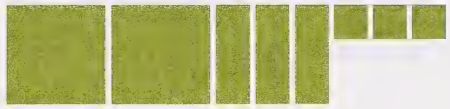
$4x^2+8$



Combine the two sets of tiles, and simplify by removing the zero pairs.



This is the result.



$\therefore\ -2x^2+3x-5+\left(4x^2+8\right)=2x^2+3x+3$

Method 2: Using Paper and Pencil

$$\begin{array}{r} -2x^2+3x-5 \\ +\ 4\left(x^2+8\right) \\ \hline \end{array} \Rightarrow \begin{array}{r} -2x^2+3x-5 \\ +\left(4x^2\quad\ +8\right) \\ \hline 2x^2+3x+3 \end{array}$$

Notice that the like terms are aligned.

1. Add the polynomials by modelling; then add using paper and pencil.

a. $(2x^2+3x-5)+(3x^2-x+2)$

b. $(3y^2-4y+1)+(-2y^2+3y+4)$

c. $(4a^2-5a)+(2a^2+3a-2)$

d. $(y^2+6y+5)+(3y^2-8)$

e. $(-3b^2+5b+1)+(b^2-3b+4)$

2. Add the following polynomials.

a.
$$\begin{array}{r} 2x^2+7 \\ +\ (3x^2-4) \\ \hline \end{array}$$

b.
$$\begin{array}{r} 3a^2+5a+6 \\ +\ (-4a^2+2a-3) \\ \hline \end{array}$$

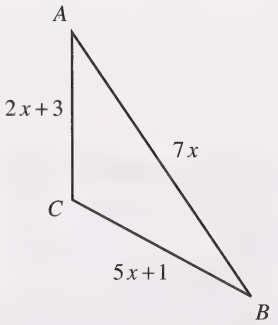
c.
$$\begin{array}{r} 8n^2-5n+7 \\ +\ (-3n^2-4) \\ \hline \end{array}$$

d.
$$\begin{array}{r} 3m^3+2m^2-4m+8 \\ +\ (m^3+7m-6) \\ \hline \end{array}$$

3. a. Write an expression for the perimeter of ΔABC.

b. Write the expression in simplest form.

c. If $x = 3$ cm, what is the perimeter?



4. The perimeter of a plot of land is given by the expression $(2x+3)+(3x-2)+(x+8)$.

a. Write the expression in simplest form.

b. If $x = 5$ m, what is the perimeter of the plot?

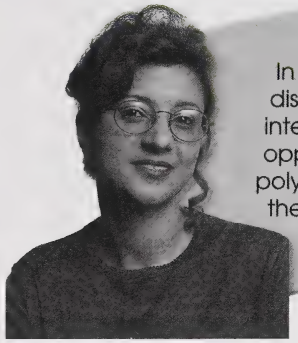
c. What shape is the plot of land when $x = 5$ m?

5. a. Add $(4x^2+3x-2)$ and (x^2-x+5).

b. Check your answer to question 5.a. by subsituting 2 for x.

Check your answers by turning to the Appendix, page 102.

Subtracting Polynomials



In your work with integers, you discovered that subtracting an integer is the same as adding its opposite. Similarly, subtracting a polynomial is the same as adding the opposite of the polynomial.

Example

Find the difference between the following polynomials:

$$\left(x^2+3x-2\right)-\left(x^2+2x+1\right)$$

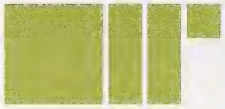
Solution

Method 1: Using Models

Model each polynomial.

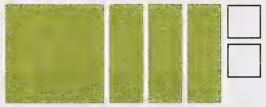
x^2+3x-2

x^2+2x+1

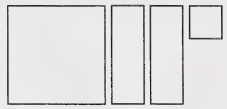


Because subtracting x^2+2x+1 is the same as adding its opposite, replace each of the tiles representing x^2+2x+1 with its opposite.

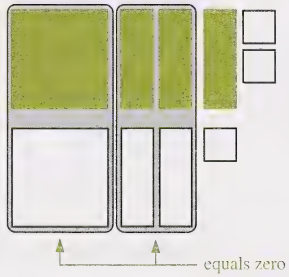
x^2+3x-2



$-x^2-2x-1$



Combine the two sets of tiles, and simplify by removing zero pairs.



This is the result.

$\therefore \left(x^2+3x-2\right)-\left(x^2+2x+1\right)=x-3$

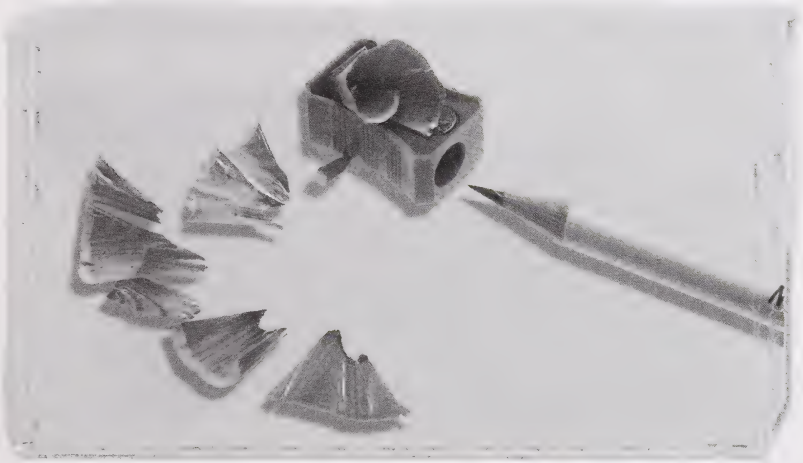
Method 2: Using Paper and Pencil

First, change the subtraction to addition.

$$\left(x^2+3x-2\right)-\left(x^2+2x+1\right)=\left(x^2+3x-2\right)+\left(-x^2-2x-1\right)$$

Then remove the parentheses, and combine the like terms.

$$\begin{aligned}\left(x^2+3x-2\right)+\left(-x^2-2x-1\right) &= x^2+3x-2-x^2-2x-1\\ &= x^2-x^2+3x-2x-2-1\\ &= x-3\end{aligned}$$



Example

Find the difference between the following polynomials:

$$\begin{array}{rl} & -3x^2+2x-1\\ - & \left(2x^2-x\right)\\ \hline \end{array}$$

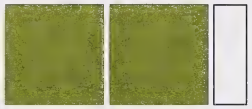
Solution

Method 1: Using Models

Model each polynomial.

$-3x^2+2x-1$

$2x^2-x$



Because subtracting $2x^2 - x$ is the same as adding its opposite, replace each of the tiles representing $2x^2 - x$ with its opposite.

$-3x^2 + 2x - 1$

$-2x^2 + x$

Combine the two sets of tiles, and simplify. **Note:** In this case there are no zero pairs to remove.

$\therefore \left(-3x^2 + 2x - 1\right) - \left(2x^2 - x\right) = -5x^2 + 3x - 1$

Method 2: Using Paper and Pencil

$$\begin{array}{r} -3x^2 + 2x - 1 \\ -\ \left(2x^2 - x\right) \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{r} -3x^2 + 2x - 1 \\ +\ \left(-2x^2 + x\right) \\ \hline -5x^2 + 3x - 1 \end{array}$$

6. Find the difference between the following polynomials using algebra tiles. Remember to change those tiles for the polynomial being subtracted to their opposites. Show the equivalent algebraic solution for each.

a. $\left(2x^2 + 3x + 4\right) - \left(x^2 + x + 4\right)$

b. $\left(x^2 - x - 4\right) - \left(2x^2 + x - 1\right)$

c. $\left(-2x^2 + x - 2\right) - \left(x^2 + 3x + 4\right)$

d. $\left(x^2 + 2\right) - \left(-2x^2 + 4x\right)$

7. Subtract the following polynomials.

a. $\left(4y^2 + 3y + 5\right) - \left(y^2 + 3y + 2\right)$

b. $\left(-3y^2 - y + 7\right) - \left(2y^2 - 3y + 3\right)$

c. $\left(3y^3 + 3y^2 - 2y + 5\right) - \left(y^3 - 2y^2 + 4y - 2\right)$

d. $\left(y^2 - 4y + 5\right) - \left(y^2 + 6\right)$

8. Subtract the following polynomials.

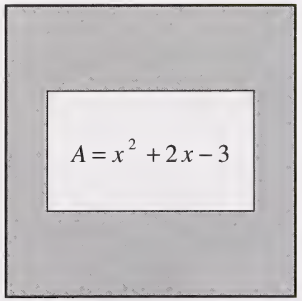
a.
$$\begin{array}{r} 4a^2 + 3a + 5 \\ -\ \left(2a^2 + 2a + 3\right) \\ \hline \end{array}$$

b.
$$\begin{array}{r} 2b^2 + 7b - 3 \\ -\ \left(b^2 - 2b - 1\right) \\ \hline \end{array}$$

c. $-6c^2+5c-8$
$-\ (4c^2+7)$

d. $8n^3+4n^2-2n+7$
$-\ (2n^3-3n^2+n+2)$

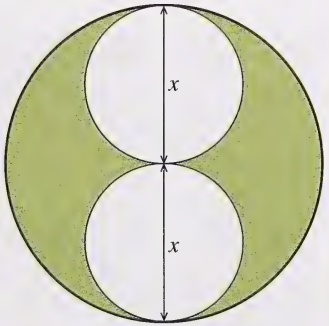
9. a. Write an expression for the area of the shaded region in the diagram if the area of the large square is $4x^2+4x+1$.



b. Write the expression in question 9.a. in simplest form.

c. If $x = 4$ cm, what is the area of the shaded region?

10. a. Write an expression that represents the area of the shaded region. (**Hint:** Area of a circle is πr^2.)



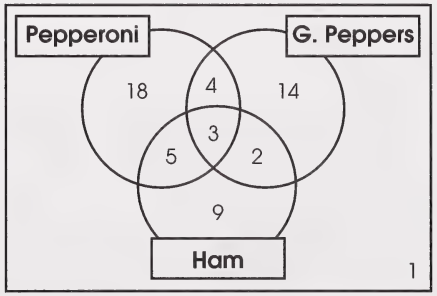
b. How does the area of the coloured region compare to the area of the two small circles?

Check your answers by turning to the Appendix, page 107.

Now Try This

Use one of the problem-solving strategies to solve the following problem.

11. The students in classes 9A and 9B at Middle Junior High School were asked whether or not they like pepperoni, green peppers, and/or ham on their pizza. The results of the 56 students surveyed are shown in the following Venn diagram.



a. How many students liked each of the following toppings?

i. pepperoni
ii. pepperoni only
iii. green peppers
iv. green peppers only
v. ham
vi. ham only
vii. pepperoni and green papers together
viii. pepperoni and ham together
ix. green peppers and ham together
x. all three toppings
xi. green peppers and pepperoni, but not ham

b. How many students did not like any of these toppings?

Check your answers by turning to the Appendix, page 112.

Looking Back

In this activity, you added and subtracted polynomials using algebra tiles and paper and pencil. When you used paper and pencil, you either wrote the addition of subtraction horizontally or in columns. As well, you examined how the operations of adding and subtracting are related, and you practised your skills with integers.

12. On a test, a student answered one of the questions as follows:

$$\begin{aligned}(2x^2 - x + 5) - (x^2 + 5x - 1) &= 2x^2 - x + 5 - x^2 + 5x - 1 \\ &= 2x^2 - x^2 - x + 5x + 5 - 1 \\ &= x^2 + 4x + 4\end{aligned}$$

a. In your journal, explain what error the student made.

b. Use algebra tiles to determine the actual difference.

c. Use paper and pencil to determine the difference.

Check your answers by turning to the Appendix, page 112.

Activity 4: Multiplying Monomials

To find the area of a rectangle, multiply the length by the width. For example, to find the area of the photograph of the Eiffel Tower, multiply 10 cm by 8 cm.

$$\begin{aligned}A &= 10 \times 8 \\ &= 80 \text{ cm}^2\end{aligned}$$

You can apply the concept of area of a rectangle to multiplying monomials by forming a rectangle with the algebra tiles that represent the product of the two monomials.

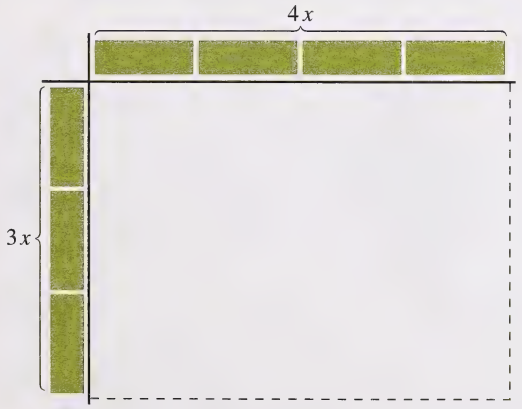
Example

Find the product of $(3x)(4x)$.

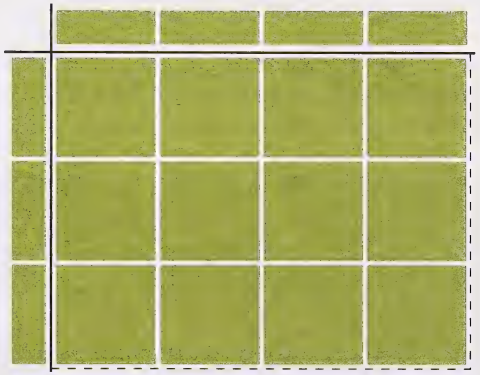
Solution

Method 1: Using Models

Use the sides of the algebra tiles to mark off the dimensions of a rectangle.



Use the marks as guides to make a rectangle with algebra tiles. Use this rectangle to determine the product.



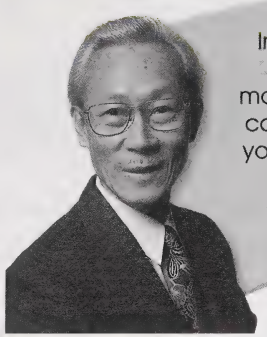
There are 12 x^2-tiles in the rectangle.

$\therefore\ (3x)(4x) = 12x^2$

Method 2: Using Paper and Pencil

$$(3x)(4x) = (3)(4)(x)(x)$$
$$= 12x^2$$

Multiply the numerical coefficients; then multiply the literal coefficients.



In the preceding example, you modelled the product of two monomials with positive numerical coefficients. In the next example, you will model the product of two monomials with numerical coefficients that differ in sign.

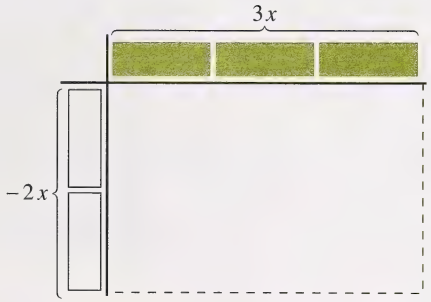
Example

Find the product of $(-2x)(3x)$.

Solution

Method 1: Using Models

Use the sides of the algebra tiles to mark off a rectangle.



Use the marks as guides to make a rectangle with algebra tiles. Use this rectangle to determine the product.

Because there is one negative factor, $(-2x)(3x)$ has the opposite product of $(2x)(3x)$.

There are six $-x^2$-tiles in the rectangle.

$\therefore\ (-2x)(3x) = -6x^2$

Method 2: Paper and Pencil

$$(-2x)(4x) = (-2)(3)(x)(x)$$
$$= -6x^2$$

Multiply the numerical coefficients; then multiply the literal coefficients.

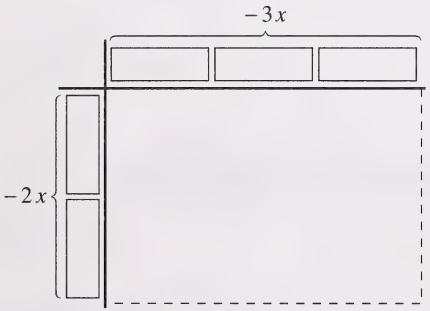
Example

Find the product of $(-2x)(-3x)$.

Solution

Method 1: Using Models

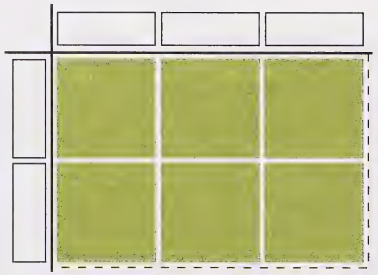
Use the sides of the algebra tiles to mark off the dimensions of a rectangle.



Use the marks as guides to make a rectangle with algebra tiles. Use this rectangle to determine the product.

Because there are two negative factors, $(-2x)(-3x)$ has the same product as $(2x)(3x)$.

There are six x^2-tiles in the rectangle.



$\therefore\ (-2x)(-3x) = 6x^2$

Method 2: Using Paper and Pencil

$$(-2x)(-3x) = (-2)(-3)(x)(x)$$
$$= 6x^2$$

Multiply the numerical coefficients; then multiply the literal coefficients.

1. Find the product of each of the following by modelling; then use paper and pencil to find each product.

a. $(2x)(4x)$ **b.** $(-2x)(4x)$ **c.** $(-2x)(-4x)$

Check your answers by turning to the Appendix, page 113.

The following summarizes how to multiply two or more monomials using algebra.

To multiply two or more monomials, first multiply the numerical coefficients and then the literal coefficients.

2. Find the products of the following monomials.

a. $(2m)(2n)$

b. $(3a)(4a^3)$

c. $(6x)(3y^2)$

d. $(2x^2)(-4xy^2)$

e. $(-5ab)(-2ab)$

f. $(4x)(3xyz)$

g. $(-4a)(2b^2)$

h. $(3x^2y^2)(4y^3)$

i. $2ab \times 3ac$

j. $3a^2b^3c \times 4ab^2c$

k. $(2m)(3m)(4m^2)$

l. $(-2a^2)(3ab)(a^3b^2)$

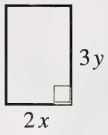
m. $(2a)(2a)$

n. $(2a)^3$

Recall that $(2a)^3 = (2a)(2a)(2a)$.

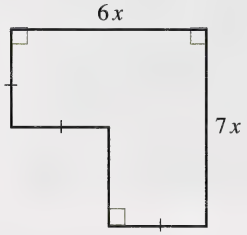
3. Write an expression for the area of each figure; then simplify.

a.

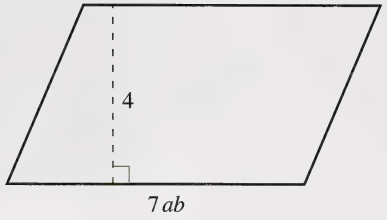


b.

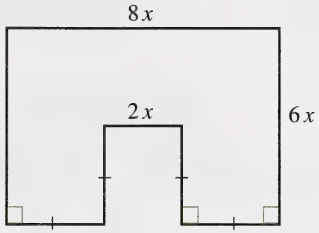
c.



d.

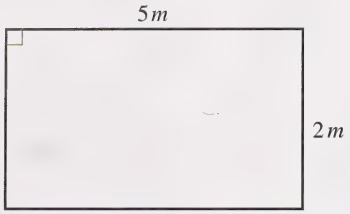


e.

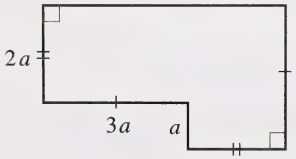


4. Find the perimeter, P, and the area, A, of the following figures.

a.

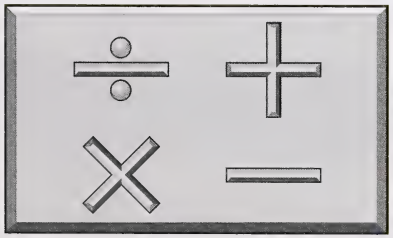


b.

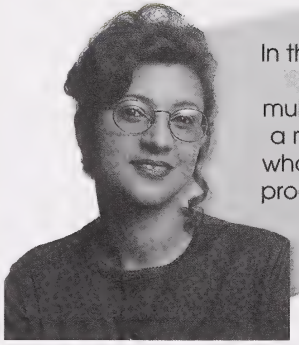


5. Express $6ab$ as a product of two monomials in as many different ways as you can. Use whole numbers only for your numerical coefficients. Consider different groupings as different ways.

Check your answers by turning to the Appendix, page 113.

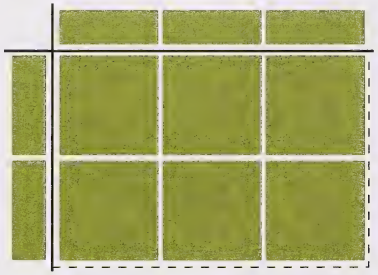


Looking Back



In this activity, you used algebra tiles to illustrate the multiplication of a monomial by a monomial. You then applied what you discovered to find the product of two monomials using paper and pencil.

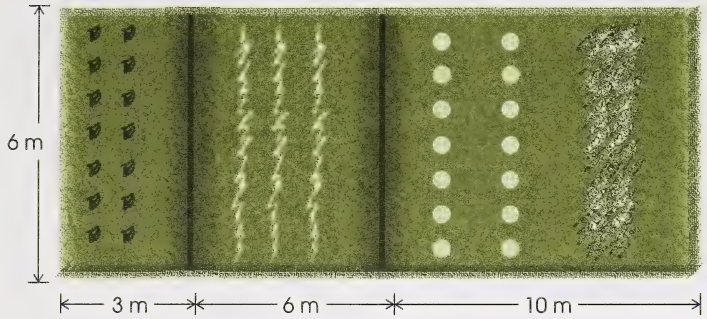
6. In your journal, copy the following diagram. If you arranged your tiles like the diagram, what product are you modelling?



Check your answer by turning to the Appendix, page 114.

Activity 5: Multiplying a Polynomial by a Monomial

The Goodstrikers have divided their garden into three different plots (as shown in the following diagram). Can you find the total area of their garden in two different ways?



Following is one way to find the area:

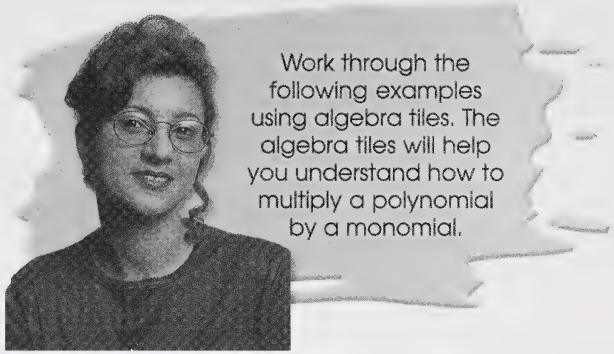
$$(6\times 3)+(6\times 6)+(6\times 10)=18+36+60$$
$$=114$$

Here is another way to find the area:

$$6(3+6+10)=6(19)$$
$$=114$$

The Goodstrkers' garden has an area of $114\ \text{m}^2$.

Multiplying a polynomial by a monomial can be similar to finding the area of the Goodstrkers' garden.


Work through the following examples using algebra tiles. The algebra tiles will help you understand how to multiply a polynomial by a monomial.

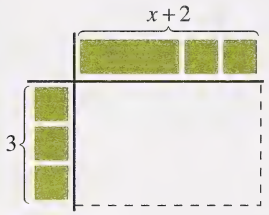
Example

Find the product of $3(x+2)$.

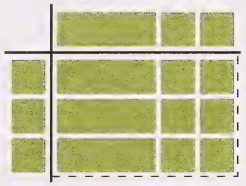
Solution

Method 1: Using Models

Use the sides of the algebra tiles to mark off the dimensions of a rectangle.

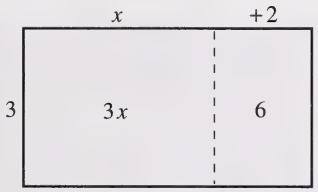


Use the marks as guides to make a rectangle with algebra tiles. Use this rectangle to determine the product.



$\therefore\ 3(x+2)=3x+6$

Note: Because one factor is a binomial, the rectangle is divided into two regions.



Method 2: Using Paper and Pencil

$$3(x+2)=3(x+2)$$
$$=3x+6$$

Multiply each term of the binomial by 3.

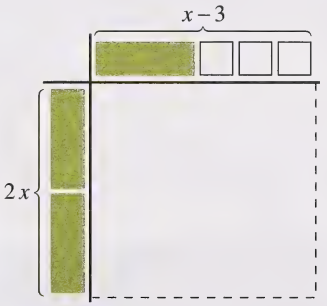
Example

Find the product of $2x(x-3)$.

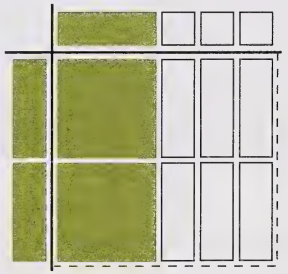
Solution

Method 1: Using Models

Use the sides of algebra tiles to mark off the dimensions of a rectangle.

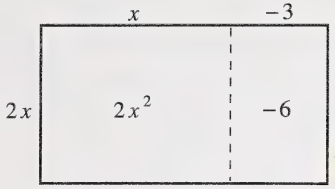


Use the marks as guides to make a rectangle with algebra tiles. Use this rectangle to determine the product.



$\therefore\ 2x(x-3)=2x-6$

Note: Because one factor is a binomial, the rectangle is divided into two regions.



Method 2: Using Paper and Pencil

$$2x(x-3)=2x(x-3)$$
$$=2x^2-6x$$

1. Use algebra tiles to find the following products.

a. $4(x+2)$

b. $x(x+3)$

c. $-x(2x-1)$

d. $2x(x-3)$

e. $3(x-1)$

f. $x(2x-2)$

Check your answers by turning to the Appendix, page 115.

The following summarizes how to multiply a polynomial by a monomial.

To multiply a polynomial by a monomial, multiply each term of the polynomial by the monomial. This process is sometimes called expanding.

Example

Find the product of $3x(x^2+2x-4)$.

Solution

Use the distributive property.

$$3x(x^2+2x-4)=3x(x^2+2x-4)$$
$$=3x^3+6x^2-12x$$

2. Determine the following products.

a. $4(x-1)$

b. $a(a+4)$

c. $5(x^2+x+1)$

d. $2b(b-3)$

e. $a(2a+3)$

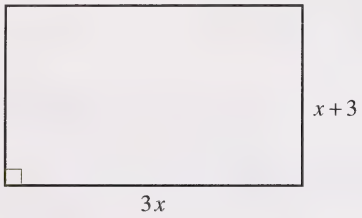
f. $2y(y^2-y+3)$

3. Simplify the following expressions.

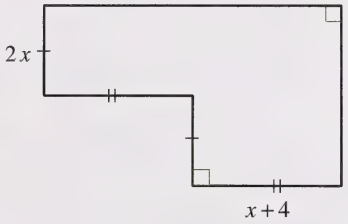
a. $2\left(a^2+2a+3\right)+3\left(a^2-a+1\right)$ **b.** $-3a\left(a^2+a-4\right)+2\left(a+1\right)$

4. Write an expression for the area of each figure. Show how you obtained your answer.

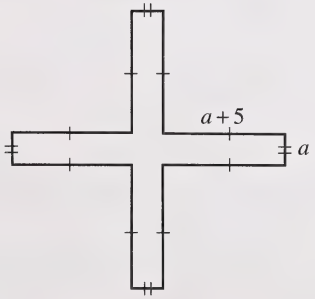
a.



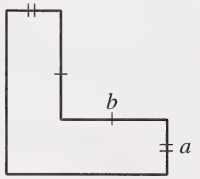
b.



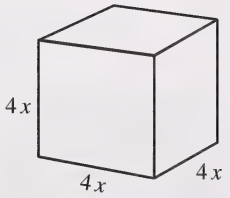
c.



5. Find the area of the following figure in two different ways.



6. Write the volume of the cube as a product of monomials; then express the answer as a single monomial.



7. **a.** If a stands for the smallest of two consecutive whole numbers, write an expression for their product.

b. Describe, in words, how you would find the product of two consecutive whole numbers using the expression from question 7.a.

c. Does the rule from question 7.b. also apply for consecutive negative integers? Explain with an example.

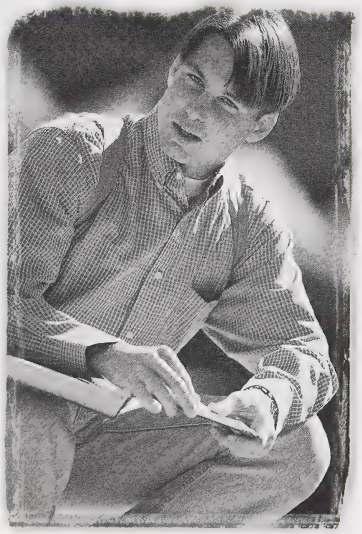
Check your answers by turning to the Appendix, page 115.

Looking Back

In this activity, you have modelled multiplication of a polynomial and a monomial with algebra tiles and you used paper and pencil to do the multiplication.

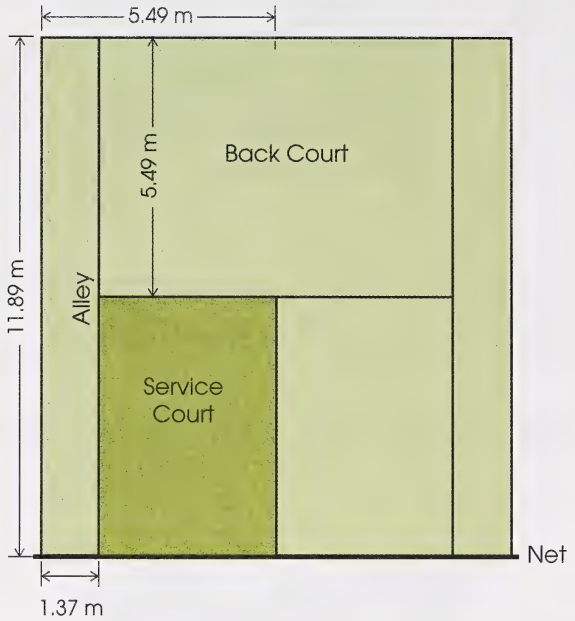
8. In your journal, draw a diagram using tiles where the area of the rectangle is $2x^2 + 4x$. Determine a possible length and width. Take your dimensions and find the product using algebra. Is your result $2x^2 + 4x$?

Check your answer by turning to the Appendix, page 117.



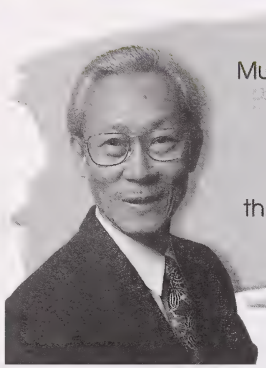
Activity 6: Multiplying Binomials

A tennis court is subdivided into several areas.



How big is the service court in tennis?

To find the area of the service court, you must find the product of $(11.89 - 5.49)(5.49 - 1.37)$. You can think of this as multiplying two binomials.



Multiplying binomials is an extension of multiplying a polynomial by a monomial. Work through the following examples.

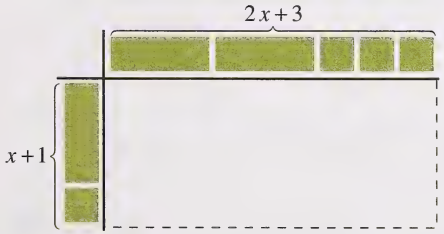
Example

Find the product of $(x+1)(2x+3)$.

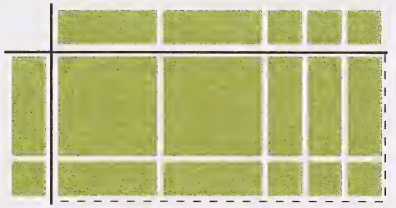
Solution

Method 1: Using Models

Use the sides of the algebra tiles to mark off the dimensions of the rectangles.

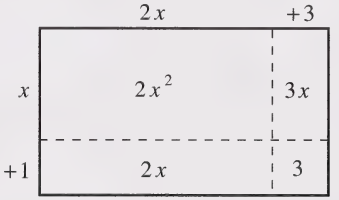


Use the marks as guides to make a rectangle. Use this rectangle to determine the product.



The product is $2x^2+5x+3$.

Note: Because each factor is a binomial, the rectangle is divided into four regions.



Method 2: Using Paper and Pencil

Multiply each term of the binomial $2x+3$ by x and then by 1.

$$(x+1)(2x+3)=(x+1)(2x+3)$$

$$=2x^2+3x+2x+3$$

$$=2x^2+5x+3$$

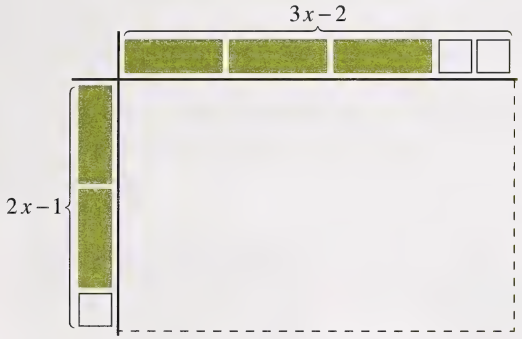
Example

Find the product of $(3x-2)(2x-1)$.

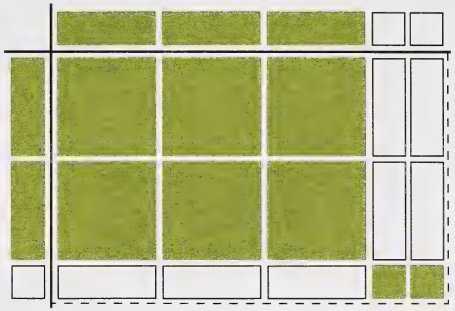
Solution

Method 1: Using Models

Use the sides of the algebra tiles to mark off the dimensions of the rectangle.

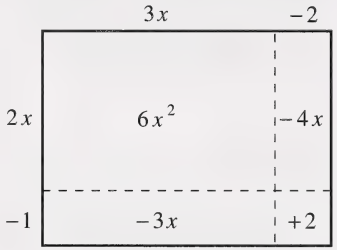


Use the marks as guides to make a rectangle. Use this rectangle to determine the product.



The product is $6x^2 - 7x + 2$.

Note: Because each factor is a binomial, the rectangle is divided into four regions.



Method 2: Using Paper and Pencil

Multiply each term of the binomial $2x-1$ by $3x$ and by -2.

$$(3x-2)(2x-1)=(3x-2)(2x-1)$$

$$=3x(2x)+3x(-1)+(-2)(2x)+(-1)(-1)$$
$$=6x^2-3x-4x+2$$
$$=6x^2-7x+2$$

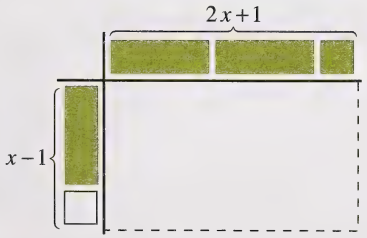
Example

Find the product of $(2x+1)(x-1)$.

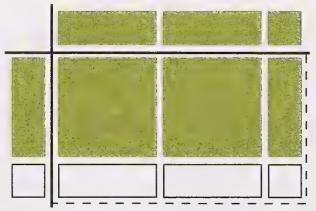
Solution

Method 1: Using Models

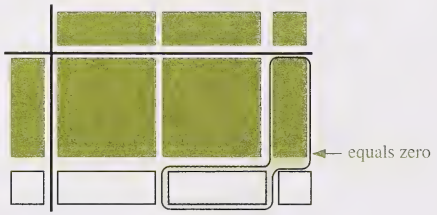
Use the sides of the algebra tiles to mark off the dimensions of the rectangle.



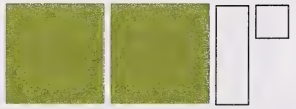
Use the marks as guides to make a rectangle.



Simplify by removing the zero pairs.



This is the result.



$\therefore\ (2x+1)(x-1)=2x^2-x-1$

Method 2: Using Paper and Pencil

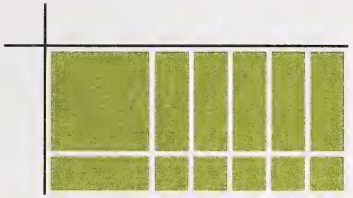
$$(2x+1)(x-1)=(2x+1)(x-1)$$
$$=2x(x)+2x(-1)+1(x)+1(-1)$$
$$=2x^2-2x+x-1$$
$$=2x^2-x-1$$

1. Use algebra tiles to find the product of each of the following.

a. $(x+3)(x+1)$ **b.** $(2x+3)(x+1)$

2. Complete the following diagrams using only positive algebra tiles; then determine the product using paper and pencil.

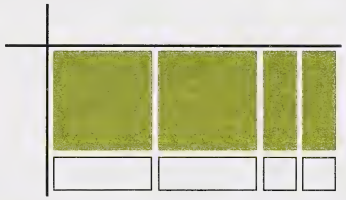
a.



b.

c.

d.



Check your answers by turning to the Appendix, page 117.

The following summarizes how to multiply two binomials.

To multiply a binomial by another binomial, multiply each term of the first binomial by each term of the second binomial; then collect the like terms. This process is sometimes called expanding.

The acronym FOIL may help you remember which terms to multiply.

F: Multiply the first terms of each binomial.
O: Multiply the two outside terms of each binomial.
I: Multiply the two inside terms of each binomial.
L: Multiply the last terms of each binomial.

Example

Expand $(y-3)(3y+5)$.

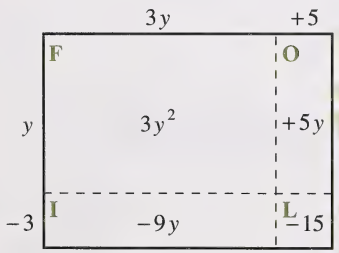
Solution

$$(y-3)(3y+5) = \underbrace{y(3y)}_{\text{F}} + \underbrace{y(5)}_{\text{O}} + \underbrace{(-3)(3y)}_{\text{I}} + \underbrace{(-3)(5)}_{\text{L}}$$

$$= 3y^2 + 5y - 9y - 15$$

$$= 3y^2 - 4y - 15$$

Note: A diagram may help you visualize the multiplication.



3. Find the product of each of the following.

a. $(x+4)(x+3)$ **b.** $(x-2)(x+5)$

c. $(2x-3)(x+4)$ **d.** $(x+6)(x-1)$

e. $(2x+6)(x+2)$ **f.** $(x-1)(x-1)$

4. a. What does n^2 mean?

b. Expand $(x+2)^2$. **Note:** $(x+2)^2 = (x+2)(x+2)$

c. Expand $(x+3)^2$.

d. The answers to 4.b. and 4.c. are **perfect square trinomials**. Is $x^2 + 8x + 16$ a perfect square trinomial? Explain.

5. Determine the following products.

a. $(x-4)(x+4)$ **b.** $(a+3)(a-3)$

c. $(2x-1)(2x+1)$ **d.** $(y-2)(y+2)$

6. a. What are the similarities in each of your answers in question 5?

b. What's special about the two terms in each product?

c. The answers to question 5 are the **differences of squares**. They come from multiplying two binomials that differ only in the sign on the second term. Would the product $(x-5)(x+5)$ give you a difference of perfect squares? Explain.

d. Use algebra tiles to illustrate the multiplication of $(x-5)$ and $(x+5)$. How do the algebra tiles help you to see the resulting product?

7. Determine the following products.

a. $(2x+3)(2x-3)$

b. $(x+4)(x-1)$

c. $(x+3)(x-5)$

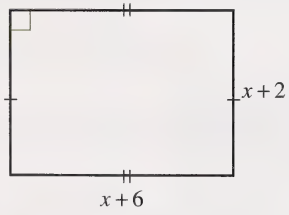
d. $(x+7)^2$

e. $(2x+5)(x+2)$

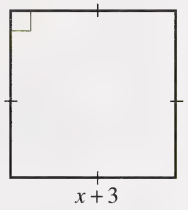
f. $(x-9)(x-6)$

8. Write an expression for the perimeter and the area of each figure; then expand each expression to find the perimeter and area.

a.



b.



Check your answers by turning to the Appendix, page 118.

Now Try This

Use problem-solving strategies to solve the following problems.

9. Which two consecutive whole numbers have squares that differ by 81? Solve this problem in two different ways, one of which must be algebraically (that is, using an equation).

10. To determine their overall standing at the end of the season, all the downhill skiers who competed were awarded 5 points for each win, 3 points for each second-place finish, and 2 points for each third-place finish. If Marten accumulated 15 points, how many possible combinations of first, second, and third places could he have achieved over the season?

Check your answers by turning to the Appendix, page 121.

Looking Back

In this activity, you manipulated algebra tiles to discover an algebraic method for multiplying binomials. You then applied this method to multiplying binomials using paper and pencil.

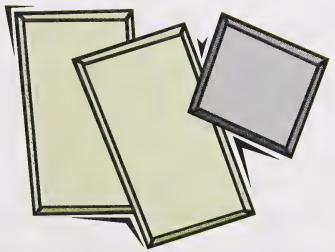
11. In your journal, do the following multiplications.

a. $(2x-1)(x+5)$ **b.** $(x+5)^2$

c. $(x-6)(x+6)$

How did you find each answer? Did you use algebra tiles or did you use pencil and paper? If you used pencil and paper, did you use any shortcuts or memory aids? Explain how you approached each question and identify the types of polynomials you obtained.

Check your answers by turning to the Appendix, page 122.



Activity 7: Dividing by a Monomial

Have you ever played three-on-three basketball or touch football? When you get together with your friends for a game of three-on-three basketball or touch football, you put yourselves into groups. Division is deciding how many groups of the divisor there are in the dividend.

Dividing by a Constant

You have been performing the operation of division with whole numbers for several years.

What does it mean when you divide 12 by 2? To divide 12 by 2, ask yourself, "How many groups of 2 are there in 12?"

$$12 = \underbrace{2+2+2+2+2+2}_{6}$$

There are six groups of 2 in 12. Therefore, $12 \div 2 = 6$.

Alternatively, to divide 12 by 2, ask yourself, "There are two groups of how many in 12?

$$12 = 6+6$$

There are two groups of 6 in 12. Therefore, $12 \div 2 = 6$.

You follow the same procedure when dividing a polynomial by a constant.

Work through the following examples.

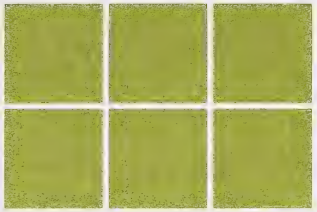
Example

Find the quotient of $6x^2 \div 3$.

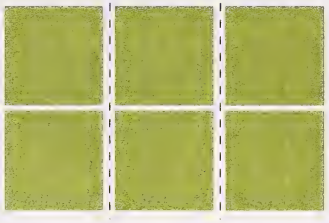
Solution

Method 1: Using Models

Use tiles to represent $6x^2$.



Divide the tiles into three equal groups. How many tiles are there in each group?



There are two x^2-tiles in each group. Therefore, $6x^2 \div 3 = 2x^2$.

Method 2: Using Paper and Pencil

$$6x^2 \div 3 = \frac{6x^2}{3}$$
$$= 2x^2$$

Example

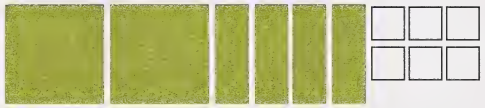
Find the quotient of the following:

$$\left(2x^2 + 4x - 6\right) \div 2$$

Solution

Method 1: Using Models

Use tiles to represent $2x^2 + 4x - 6$.



Divide the tiles into two equal groups. How many tiles are there in each group?

There are one x^2-tile, two x-tiles, and three -1-tiles in each group.

Therefore, $(2x^2+4x-6)\div 2 = x^2+2x-3$.

Method 2: Using Paper and Pencil

$$\frac{2x^2+4x-6}{2} = \frac{2x^2}{2}+\frac{4x}{2}+\frac{-6}{2}$$
$$= x^2+2x-3$$

1. Use algebra tiles to perform each of the following divisions.

a. $8x^2 \div 2$

b. $6x \div 3$

c. $(2x^2+4)\div 2$

d. $(-2x^2-6x+4)\div 2$

e. $(9x+6)\div 3$

f. $(4x^2+12x-8)\div 4$

2. Describe a rule that you could use to divide a polynomial by a constant without using algebra tiles.

3. Use the rule from question 3, to perform the following divisions.

a. $(6x-9)\div 3$

b. $(4x^2+2x-2)\div 2$

c. $\dfrac{6x^2+9x-3}{3}$

4. The perimeter of the square on the right is given.

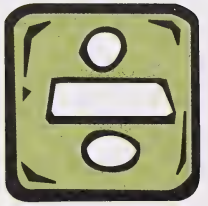
$P = 12y-8$

a. Find the length of a side.

b. Find the area.

Check your answers by turning to the Appendix, page 122.

Dividing by a Monomial



If you were asked to divide $(4x^2+2x)$ by $2x$, you would be unable to use the grouping model that was used when dividing by constants. But, since division is the opposite of multiplication, you can use the area model used for multiplying.

Work through the following example.

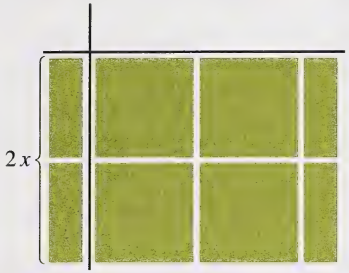
Example

Find the quotient of $(4x^2+2x)\div 2x$.

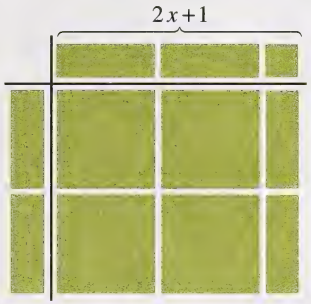
Solution

Method 1: Using Models

Arrange the algebra tiles that represent $4x^2 + 2x$ into a rectangle with a width of $2x$.



Find the length of the rectangle.



The length of the rectangle is $2x + 1$.

Therefore, $\left(4x^2 + 2x\right) \div 2x = 2x + 1$.

Method 2: Using Paper and Pencil

$$\left(4x^2 + 2x\right) \div 2x = \frac{4x^2}{2x} + \frac{2x}{2x}$$
$$= 2x + 1$$

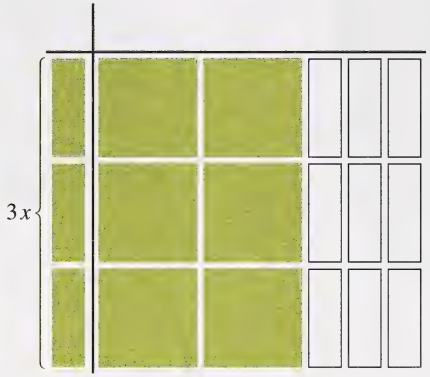
Example

Find the quotient of $\left(6x^2 - 9x\right) \div 3x$.

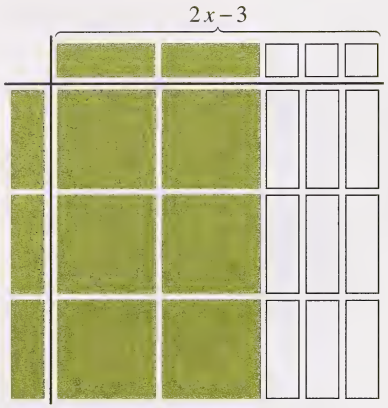
Solution

Method 1: Using Models

Arrange the algebra tiles that represent $6x^2 - 9x$ into a rectangle with a width of $3x$.



Find the length of the rectangle.



The length of the rectangle is $2x-3$.

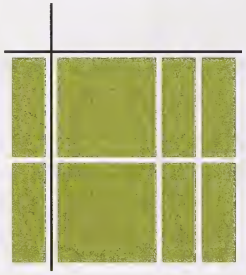
Therefore, $\left(6x^2-9x\right)\div 3x=2x-3$.

Method 2: Using Paper and Pencil

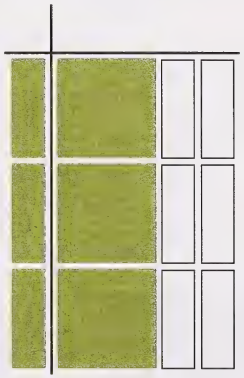
$$\left(6x^2-9x\right)\div 3x=\frac{6x^2}{3x}-\frac{9x}{3x}$$
$$=2x-3$$

5. Use the rectangular display of algebra tiles to do the indicated division.

a. $\left(2x^2+4x\right)\div 2x$

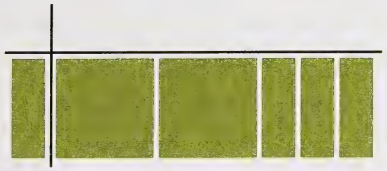


b. $\left(3x^2-6x\right)\div 3x$



c. $\left(6x^2-2x\right)\div 2x$

d. $\left(2x^2+3x\right)\div x$



Check your answers by turning to the Appendix, page 124.

The following summarizes how to divide a polynomial by a monomial.

To divide a polynomial by a monomial, divide each term of the polynomial by the monomial. That is, divide the numerical coefficients and the literal coefficients each time.

Work through the following examples to see how the division of a polynomial by a monomial can be performed using paper and pencil.

Example

Divide $\left(8x^2+6x\right)$ by $2x$.

Solution

$$\left(8x^2+6x\right)\div 2x=\frac{8x^2}{2x}+\frac{6x}{2x}$$
$$=4x+3$$

Example

Divide $\left(12x^3+8x^2-4x\right)$ by $4x$.

Solution

$$\frac{12x^3+8x^2-4x}{4x}=\frac{12x^3}{4x}+\frac{8x^2}{4x}+\frac{-4x}{4x}$$
$$=3x^2+2x-1$$

Remember: A negative term divided by a positive term gives a negative term.

6. Use algebra tiles to do the following divisions.

a. $(6x^2 - 4x) \div 2x$

b. $(6x^2 + 3x) \div 3x$

c. $(2x^2 + 6x) \div 2x$

d. $(3x^2 + 6x) \div 3x$

7. Use paper and pencil to do the following divisions.

a. $(6x^2 + 5x) \div x$

b. $(4x^2 + 6x) \div 2x$

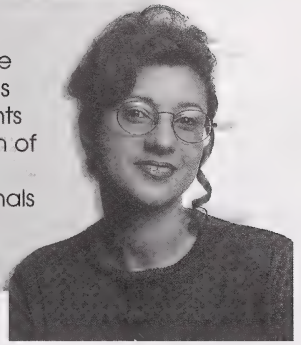
c. $(10x^2 + 5x) \div 5x$

d. $\dfrac{4x^2 + 2x + 6}{2}$

e. $\dfrac{3x^2 - 6x}{3x}$

f. $\dfrac{5x^3 + 10x^2 - 15x}{5x}$

Check your answers by turning to the Appendix, page 125.



Up to this point, all of the examples and questions have involved coefficients that divide evenly. Division of monomials can be performed over the rationals as well.

Example

Divide $(3m^3 + 8m^2 - 5m)$ by $4m$.

Solution

$$\frac{3m^3 + 8m^2 - 5m}{4m} = \frac{3m^3}{4m} + \frac{8m^2}{4m} + \frac{-5m}{4m}$$

$$= \frac{3m^2}{4} + 2m - \frac{5}{4}$$

$$= \frac{3}{4}m^2 + 2m - \frac{5}{4}$$

8. Perform the following divisions. Verify your answer by multiplying.

a. $\dfrac{4x^2 + 6x - 8}{5}$

b. $\dfrac{8x^3 + 10x^2 - 4x}{-4x}$

9. Perform the following divisions.

a. $\left(6x^3+4x^2+8x\right)\div 3x$

b. $\left(12x^4+8x^3-16x^2+4x\right)\div\left(-4x\right)$

c. $\left(x^3-9x\right)\div x$

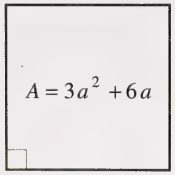
d. $\dfrac{-12y^3+9y^2-y}{-3y}$

e. $\dfrac{24-8m+4m^2}{4}$

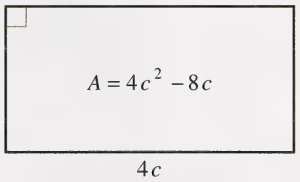
f. $\dfrac{14a^2b^3+7a^2b^2-7ab^2}{7ab}$

10. Find the width of each rectangle.

a.



b.



11. If the width of a rectangle is $2b$ and its area is $6b^2+2b$, find the following:

a. the length **b.** the perimeter

12. Find the value of each expression if $x=2$ and $y=-3$.

a. $\dfrac{6x^2y^2}{2xy}$ **b.** $3xy$

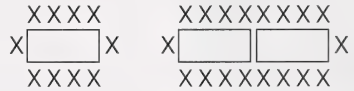
13. What do you notice about your answers in question 12? Explain.

Check your answers by turning to the Appendix, page 126.

Now Try This

Use one of the problem-solving strategies to solve the following pattern.

14. A banquet table can seat four people on each side and one person on each end. If 13 of these tables are put together in one long row, what is the maximum number of people that can be seated?

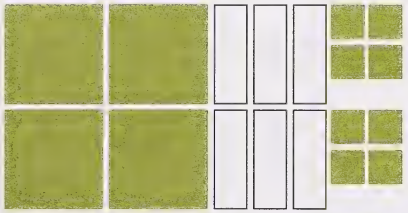


Check your answer by turning to the Appendix, page 128.

Looking Back

In this activity, you divided a polynomial by a monomial using algebra tiles and paper and pencil.

15. In your journal, copy the following diagram of algebra tiles.



Answer the following questions.

a. What polynomial is represented by the tiles?

b. Explain why the polynomial is divisible by 2 but not divisible by 4 over the integers.

c. What is the quotient when the polynomial modelled is divided by 2?

Check your answers by turning to the Appendix, page 128.

Conclusion

In this section, you identified some terminology associated with polynomials and developed some principles on how to model polynomials. You then modelled the four basic operations—addition, subtraction, multiplication, and division—with polynomials using algebra tiles. You also performed these four operations with polynomials using paper and pencil.

Just as a construction worker builds a solid foundation and framework before doing the finishing work on a house, you have now completed the foundation and framework of your knowledge of polynomials. Are you ready to start some finishing work with polynomials in Section 2?

Assignment

Turn to Assignment Booklet 4A and complete the assignment for Section 1.

SECTION 2

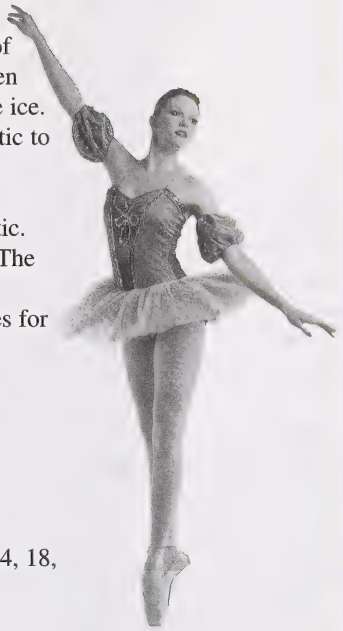
Factoring

Have you ever had to ask someone for help to find a particular address in a city with named streets? You may have found yourself going down the wrong streets and may have even ended up back where you started. But if you had patience and determination, looked for clues, and learned from your mistakes, you likely were successful.

How do you find your way back to the starting point once you've successfully negotiated your way through? Well, if you've remembered the route you took to get there, you can retrace your steps by doing everything in reverse.

This is not unlike the situation you find yourself in at this point in learning about polynomials. You've successfully found your way through the maze of polynomials—learning the correct paths and procedures to follow in order to add, subtract, multiply, and divide polynomials. Now you are being asked to factor polynomials. If you remember the procedures you followed in multiplying polynomials, you should be able to factor polynomials successfully. You are simply going to reverse the procedure, like finding your way back to your starting point.

Activity 1: Common Factors



Many skills can be transferred from one area of life to another. For example, figure skaters often transfer skills they have learned in ballet to the ice. Similarly, you can transfer skills from arithmetic to algebra.

In elementary school, you focused on arithmetic. In high school, your focus will be on algebra. The rules and procedures for arithmetic can be generalized to develop the rules and procedures for algebra.

Work through the following examples.

Example

Find the greatest common factor (GCF) of 24, 18, and 30.

Solution

Step 1: Factor each number.

$$24 = 2 \times 2 \times 2 \times 3$$
$$18 = 2 \times 3 \times 3$$
$$30 = 2 \times 3 \times 5$$

Step 2: Find the common factors, and multiply to find the GCF.

$$24 = 2 \times 2 \times 2 \times 3$$
$$18 = 2 \times 3 \times 3$$
$$30 = 2 \times 3 \times 5$$

$$\therefore \text{GCF} = 2 \times 3$$
$$= 6$$

Example

What is the GCF of $2x^3$, $8x^2$, and $-4x$?

Solution

Step 1: Factor each term.

$$2x^3 = 2 \times x \times x \times x$$
$$8x^2 = 2 \times 2 \times 2 \times x \times x$$
$$-4x = -1 \times 2 \times 2 \times x$$

Step 2: Find the common factors, and multiply to find the GCF.

$$2x^3 = 2 \times x \times x \times x$$
$$8x^2 = 2 \times 2 \times 2 \times x \times x$$
$$-4x = -1 \times 2 \times 2 \times x$$

$$\therefore \text{GCF} = 2 \times x$$
$$= 2x$$

1. Find the greatest common factor of the following sets of numbers or terms.

a. 15, 20, 35

b. $12, -16, 20$

c. $6x, 8x^2, 4x^3$

d. $-9x, 15x^2, -12x^3$

e. $24z^4, 30z^3, -12z^2$

f. $9ab^2c, 12a^2b, 21b^2c$

Check your answers by turning to the Appendix, page 128.

You can apply what you have learned about finding the GCF to factor certain polynomials. Work through the following example.

Example

Factor $3a^2b - 6ab^2 + 9ab$ completely.

Solution

Step 1: Find the GCF of the three terms.

$$\begin{aligned} 3a^2b &= \quad 3 \times a \times a \times b \\ -6ab^2 &= -1 \times 2 \times 3 \times a \times b \times b \\ 9ab &= \quad 3 \times 3 \times a \times b \end{aligned}$$

$$\therefore \text{GCF} = 3 \times a \times b$$
$$= 3ab$$

Step 2: To find the other factor, divide the polynomial by $3ab$.

$$\frac{3a^2b - 6ab^2 + 9ab}{3ab} = \frac{3a^2b}{3ab} + \frac{-6ab^2}{3ab} + \frac{9ab}{3ab}$$
$$= a - 2b + 3$$

Therefore, $3a^2b - 6ab^2 + 9ab$ factored completely is $3ab(a - 2b + 3)$.

2. Find the missing factor.

a. $4a^2 + 6a = 2a$ ( )

b. $7a^3 - 14a^2 + 28a = 7a$ ( )

c. $18b + 3b^2 =$ $(6 + b)$

d. $12b^2 + 20b^3 - 24b^4 =$ $(3 + 5b - 6b^2)$

e. $10b^3 + 8b^2 - 12b + 6 = 2$ ( )

3. Factor each of the following. Check your answers by expanding.

a. $2x^2+8x$

b. $4ab^2-7a^2b$

c. $4a^2+6a+8$

d. $7y^2+14y^3$

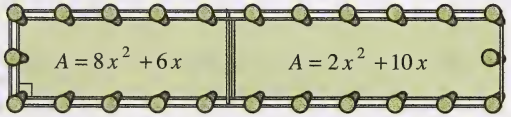
e. $4a-16$

f. $2xy+6x^2y^2-8x^3y^3$

g. $ab+ac$

h. $ab+ac-2ad+4ae$

4. Jeffrey and Jamel have a hobby farm on which they raise ostriches. Jeffrey and Jamel have two attached rectangular pens as shown in the diagram. Find the length of the common side.



5. The formula for the perimeter of a rectangle is $P=2\ell+2w$. Factor this formula.

Check your answers by turning to the Appendix, page 129.

Common Binomial Factors

Sometimes the greatest common factor is a binomial. The method used to factor these expressions is very similar to the method used to factor expressions with common monomial factors.

Example

Factor $x(x+3)+2(x+3)$.

Solution

The binomial $x+3$ is a common factor in each term. It is multiplied by x and by 2.

$$\therefore\ x(x+3)+2(x+3)=(x+3)(x+2)$$

Check

LS	RS
$x(x+3)+2(x+3)$	$(x+3)(x+2)$
$=x^2+3x+2x+6$	$=x(x)+x(2)+3(x)+3(2)$
$=x^2+5x+6$	$=x^2+2x+3x+6$
	$=x^2+5x+6$

LS = RS

Example

Factor $a(a-2)-b(a-2)$.

Solution

The common binomial factor is $a-2$.

$$\therefore\ a(a-2)-b(a-2)=(a-2)(a-b)$$

Check

Let $a=4$ and $b=5$. Substitute these values into each expression, and evaluate.

LS	RS
$a(a-2)-b(a-2)$	$(a-2)(a-b)$
$=4(4-2)-5(4-2)$	$=(4-2)(4-5)$
$=4(2)-5(2)$	$=(2)(-1)$
$=8-10$	$=-2$
$=-2$	

LS = RS

6. Identify the common binomial factor in each of the following expressions.

a. $(m+n)a+(m+n)b$

b. $3c(a-3)+2(a-3)$

c. $y(2x+3)+1(2x+3)$

d. $(d-1)x+(d-1)y$

e. $a(a+3)-b(a+3)$

f. $y(x-1)-3(x-1)$

7. Find the common binomial factor of each expression, and factor completely.

a. $x(x-7)+3(x-7)$

b. $2y(y+3)-1(y+3)$

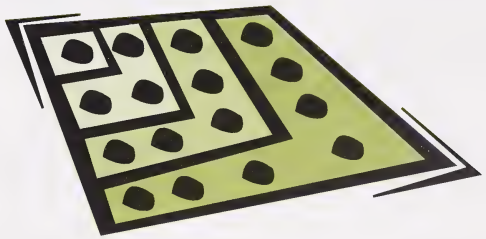
c. $a(a+7)+3(a+7)$

d. $(2n+3)n+(2n+3)3$

e. $(m+1)m-(m+1)3$

f. $2t(t+3)+5(t+3)$

Check your answers by turning to the Appendix, page 131.

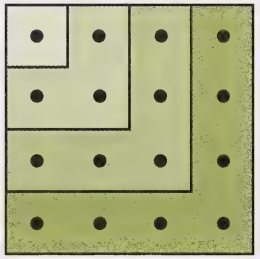


Now Try This

Being able to recognize patterns is an important skill in factoring polynomials. The following problem involves recognizing member patterns and using these patterns to make generalizations.

8. a. Find the sum of $1+3+5+7$.

b. Using the pattern suggested by the following diagram, find the sum of $1+3+5+7+\ldots+15$.



c. What is the sum of the first twenty odd numbers?

d. If n represents the number of odd numbers being added, write an expression for the last odd number.

e. Find the rule for the sum of $1+3+5+7+\ldots+(2n-1)$.

Check your answers by turning to the Appendix, page 131.

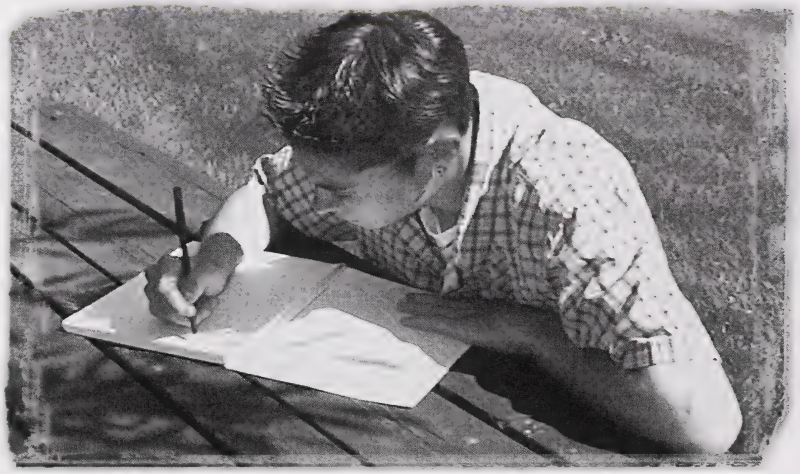
Looking Back

In this activity, you extended your knowledge of factoring in arithmetic to factoring polynomials with common factors that were either monomials or binomials.

9. a. In your journal, write one binomial that has a monomial as a common factor.

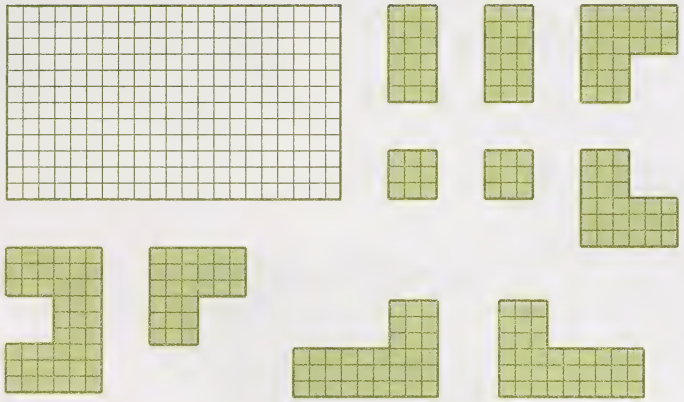
b. You have just completed exercises where there were common factors. This is not always the case. Provide an example of a binomial that has no integer that is a common factor except 1 or -1.

Check your answers by turning to the Appendix, page 131.



Activity 2: Factoring Trinomials of the Form $x^2 + bx + c$

At the back of the Appendix, find the page with the following rectangular grid and pieces; then cut out the grid and each piece. If you are photocopying this page, you may wish to glue it to heavier paper or boxboard before cutting out the pieces. This will make them easier to handle.



Can you arrange all of the pieces given to fill the rectangular grid?

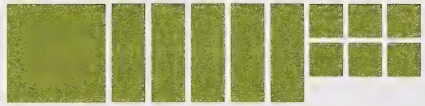
Solving this puzzle is similar to the process used in factoring trinomials using algebra tiles. You use the algebra tiles to represent the trinomial and then arranged the tiles to form a rectangle. The factors of the trinomial are the lengths of the sides of the rectangle.

Example

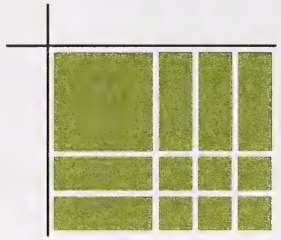
Factor the trinomial $x^2 + 5x + 6$.

Solution

Step 1: Model the trinomial.

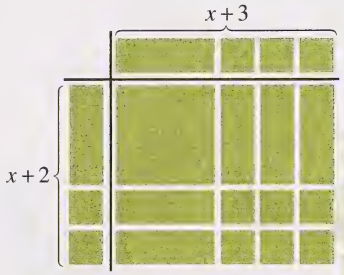


Step 2: Arrange the tiles to form a rectangle. Place the x^2-tiles on the upper left and the 1-tiles on the lower right.



This format is the standard way of arranging tiles.

Step 3: Find the factors of the trinomial represented by the length and width of the rectangle.



$\therefore\ x^2 + 5x + 6 = (x+2)(x+3)$

Example

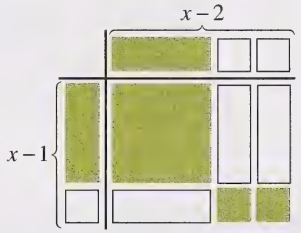
Factor the trinomial $x^2 - 3x + 2$.

Solution

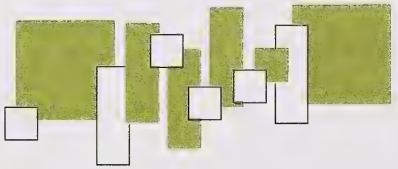
Step 1: Model the trinomial.

Step 2: Arrange the tiles to form a rectangle.

Step 3: Place the tiles representing the length and the width.



$\therefore\ x^2 - 3x + 2 = (x-2)(x-1)$



In the preceding examples, you constructed rectangles with just the tiles needed to model the original trinomial. In the next example, you will have to add additional tiles, tiles that form zero pairs, to complete the rectangle.

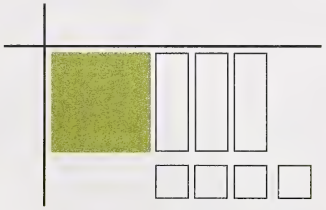
Example

Factor the trinomial $x^2 - 3x - 4$.

Solution

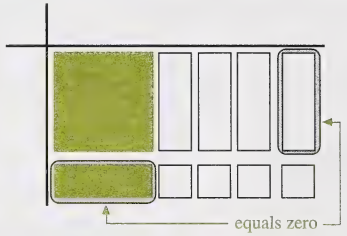
Step 1: Model the trinomial.

Step 2: Arrange the tiles to form a rectangle.

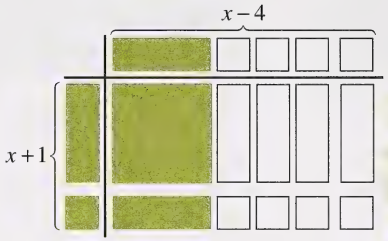


Step 3: Notice that it is impossible to arrange the tiles to form a rectangle without using more tiles. You can add more tiles without changing the trinomial by applying the zero principle.

Add one x-tile and one $-x$-tile to complete the rectangle.



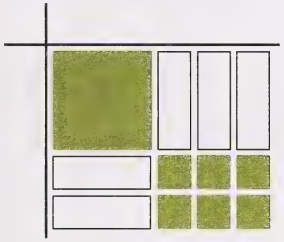
Step 4: Place the tiles representing the length and width.



$\therefore\ x^2 - 3x - 4 = (x+1)(x-4)$

1. a. Write the trinomial represented by the rectangular model. Then use the rectangular model to factor the trinomial.

b. Write the trinomial represented by the rectangular model. Then use the rectangular model to factor the trinomial.



c. Write the trinomial represented by the model. Add tiles to complete the rectangle. Then use the rectangular model to factor the trinomial.

2. Use algebra tiles to factor the following trinomials.

a. $x^2 + 8x + 12$

b. $x^2 + 8x + 15$

c. $x^2 - 5x + 4$

d. $x^2 - 6x + 8$

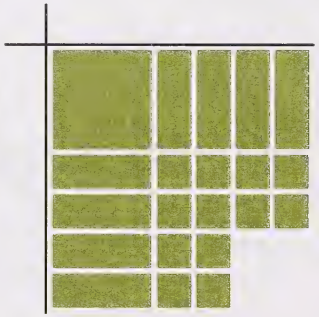
e. $x^2 - 4x - 5$

f. $x^2 + 2x - 8$

3. Annette and Krish are using algebra tiles to factor the polynomial $x^2 + 8x + 12$. Their arrangements are as follows:

Annette's

Krish's



Explain why Annette's arrangement is correct and Krish's is not.

Check your answers by turning to the Appendix, page 131.

Inspect the binomial factors of a trinomial and you should be able to see some patterns that will allow you to factor a trinomial without having to use algebra tiles.

4. Look at the following three trinomials and their factors, and note the pattern. What pattern do you notice?

$$n^2 + 7n + 12 = (n+4)(n+3)$$

$+7 = 4+3$ sum

$+12 = 4\times 3$ product

$$n^2 - 6n + 5 = (n-5)(n-1)$$

$-6 = (-5)+(-1)$ sum

$+5 = (-5)\times(-1)$ product

$$n^2 + 3n - 10 = (n+5)(n-2)$$

$+3 = 5+(-2)$ sum

$-10 = 5\times(-2)$ product

Check your answer by turning to the Appendix, page 134.

To factor a polynomial of the form $x^2 + bx + c$, you need to find two numbers whose product is c and whose sum is b.

For instance, to factor a trinomial like $x^2 + 10x + 16$, you need to find two numbers whose product is 16 and whose sum is 10.

5. a. Write out all the factors of 16.

b. Which pair of factors add up to 10?

Check your answers by turning to the Appendix, page 134.

The numbers 8 and 2 become the second terms in the factors of the trinomial $x^2 + 10x + 16$. Thus, the factors are $x+8$ and $x+2$.

You can check the factors by multiplying.

$$\begin{aligned}(x+8)(x+2) &= x^2 + 2x + 8x + 16\\ &= x^2 + 10x + 16\end{aligned}$$

$$\therefore\ x^2 + 10x + 16 = (x+8)(x+2)$$

Example

Factor the trinomial $y^2 - y - 20$.

Solution

You need to find two numbers whose product is -20 and whose sum is -1. The factors of -20 are as follows:

- -20 and 1 ← The sum is -19.
- 20 and -1 ← The sum is 19.
- -10 and 2 ← The sum is -8.
- 10 and -2 ← The sum is 8.
- -5 and 4 ← The sum is -1.
- 5 and -4 ← The sum is 1.

The correct numbers are -5 and 4, thus becoming the second terms in the binomial factors. The factors are $y-5$ and $y+4$.

$\therefore\ y^2 - y - 20 = (y-5)(y+4)$

Check

$$(y-5)(y+4) = y^2 + 4y - 5y - 20$$
$$= y^2 - y - 20$$

6. Copy and complete the factoring of the following trinomials by filling in the proper sign ($-$ or $+$) in the binomial factors. Check your answers.

a. $x^2 + 11x + 30 = (x \square 5)(x \square 6)$

b. $x^2 - 9x + 20 = (x \square 5)(x \square 4)$

c. $n^2 - 3n - 18 = (n \square 6)(n \square 3)$

d. $y^2 + 5y - 14 = (y \square 7)(y \square 2)$

7. Find the missing binomial factor for each of the following.

a. $x^2 + 13x + 36 = (x+4)(\quad)$

b. $x^2 - 14x - 32 = (x-16)(\quad)$

c. $y^2 - 11y + 24 = (y-3)(\quad)$

8. Factor the following trinomials. Check your answers.

a. $x^2 - 12x + 20$

b. $n^2 - 4n - 12$

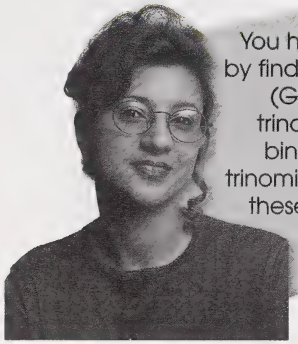
c. $y^2 + 11y - 12$

d. $m^2 - 6m + 9$

e. $p^2 + 8p + 16$

f. $b^2 + b - 56$

Check your answers by turning to the Appendix, page 134.



You have now factored polynomials by finding the greatest common factor (GCF), and you have factored trinomials into the product of two binomials. Sometimes, factoring trinomials requires you to apply both of these techniques in order to factor them completely.

Example

Factor $2x^2 + 22x + 56$.

Solution

First, factor by taking out the common factor of 2.

$$2x^2 + 22x + 56 = 2\left(x^2 + 11x + 28\right)$$

Next, factor the trinomial by finding two numbers whose product is 28 and whose sum is 11. The factors of 28 are as follows:

- 28 and 1 ← The sum is 29.
- 14 and 2 ← The sum is 16.
- 7 and 4 ← The sum is 11.

Because the sum is positive, only the positive factors of 28 are needed.

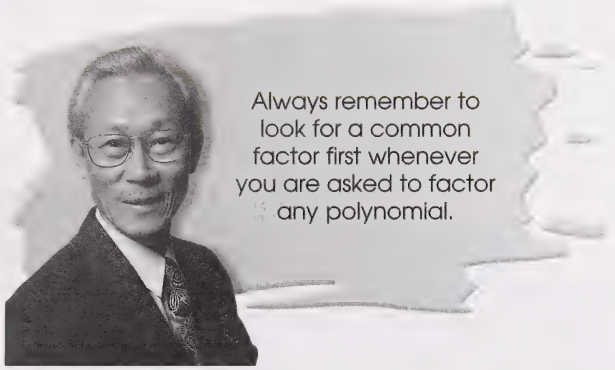
The correct numbers are 7 and 4. Thus,

the factors of $x^2 + 11x + 28$ are $(x+7)$ and $(x+4)$.

$\therefore\ 2x^2 + 22x + 56 = 2(x+7)(x+4)$

Check

$$\begin{aligned} 2(x+7)(x+4) &= 2\left(x^2 + 4x + 7x + 28\right) \\ &= 2\left(x^2 + 11x + 28\right) \\ &= 2x^2 + 22x + 56 \end{aligned}$$



9. Factor the following trinomials. **Hint:** Take out the GCF first.

a. $3x^2+15x+12$

b. $2x^2-20x+50$

c. $2x^3+14x^2+12x$

d. $3x^3+21x^2-54x$

e. $6y^2+12y+6$

f. $x^2y+3xy-40y$

g. $4x^2-28x+48$

h. x^3+2x^2+x

Check your answers by turning to the Appendix, page 135.

In Section 1, you explored special products, such as $(x+4)(x-4)$ and $(x-6)(x+6)$.

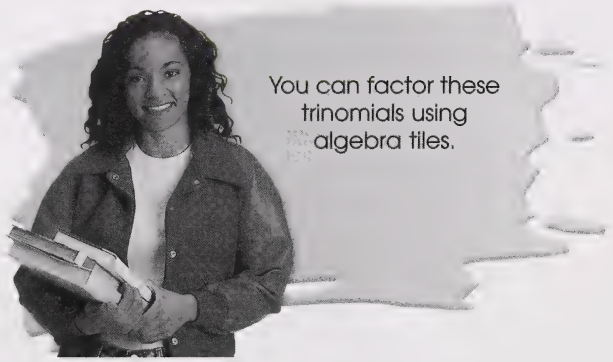
$$(x+4)(x-4)=x^2-4x+4x-16$$
$$=x^2+0x-16$$
$$=x^2-16$$

$$(x-6)(x+6)=x^2+6x-6x-36$$
$$=x^2+0x-36$$
$$=x^2-36$$

Recall that these products are called differences of squares. You can think of a difference of squares as a trinomial in which the coefficient of the middle term is 0.

$x^2-16=x^2+0x-16$ $\qquad$ $x^2-36=x^2+0x-36$

The middle term is 0.

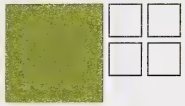


Example

Use tiles to factor $x^2 - 4$.

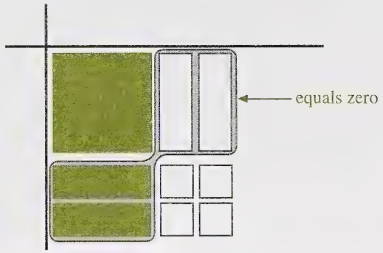
Solution

Step 1: Model the binomial.

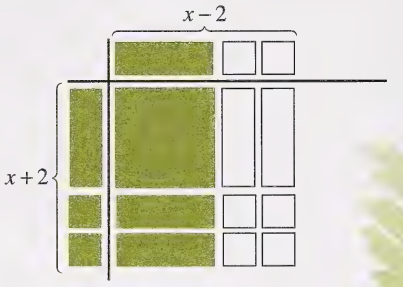


Step 2: Arrange the tiles to form a rectangle.

Step 3: Notice that it is impossible to arrange the tiles to form a rectangle without more tiles. You can add more tiles without changing the trinomial by applying the zero principle. Add two x-tiles and two $-x$-tiles to complete the rectangle.



Step 4: Place the tiles representing the length and width.

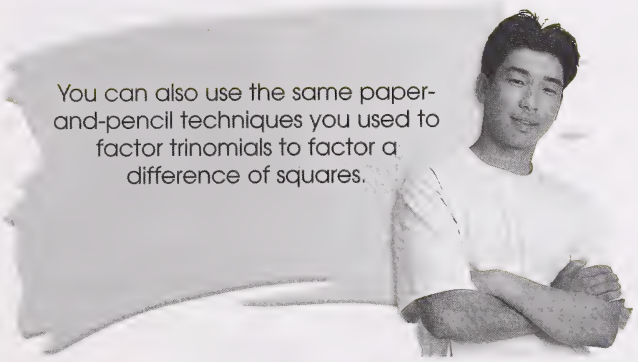


$\therefore\ x^2 - 4 = (x+2)(x-2)$

10. Use algebra tiles to factor the following binomials.

a. $x^2 - 9$ **b.** $4x^2 - 1$

Check your answers by turning to the Appendix, page 136.



Example

Factor $x^2 - 25$. Check your answer.

Solution

Write $x^2 - 25$ as a trinomial.

$$x^2 - 25 = x^2 + 0x - 25$$

You need to find two numbers whose product is -25 and whose sum is 0. The factors of -25 are as follows:

- -25 and 1 ← The sum is -24.
- -5 and 5 ← The sum is 0.
- 25 and -1 ← The sum is 24.

The correct numbers are -5 and 5, thus becoming the second terms in the binomial factors. The factors are $x-5$ and $x+5$.

$$\therefore x^2 - 25 = (x-5)(x+5)$$

Check

$$\begin{aligned}(x-5)(x+5) &= x^2 + 5x - 5x - 25 \\ &= x^2 + 0x - 25 \\ &= x^2 - 25\end{aligned}$$

11. Copy and complete the factoring of the differences of squares by filling in the proper signs in the binomial factors.

a. $x^2 - 49 = (x \square 7)(x \square 7)$

b. $y^2 - 100 = (y \square 10)(y \square 10)$

12. Find the missing binomial factor for each of the following.

a. $4n^2 - 25 = (2n+5)(\quad)$

b. $x^2 - 64 = (x-8)(\quad)$

c. $64y^2 - z^2 = (\quad)(8y+z)$

13. Factor the following binomials. Check your answers.

a. $4x^2 - 9$ **b.** $n^2 - 1$

c. $4a^2 - b^2$ **d.** $x^2 - y^2$

Check your answers by turning to the Appendix, page 136.

Did you notice a shortcut for factoring a difference of squares, such as $x^2 - 81$? To factor a difference of squares, take the square root of each term. Use a positive sign in one factor and a negative sign in the other factor.

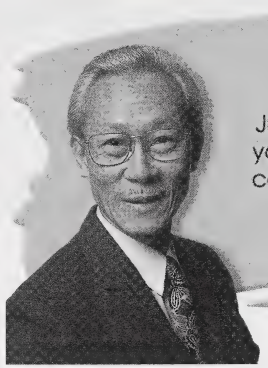
$$x^2 - 81 = (x-9)(x+9)$$

square root of 81

square root of x^2

$(x)(x) = x^2$

$(9)(9) = 81$



Just as for trinomials, you should check for common factors first.

Example

Factor $2x^2 - 32$.

Solution

First, factor by taking out the common factor of 2.

$$2x^2 - 32 = 2(x^2 - 16)$$
$$= 2(x-4)(x+4)$$

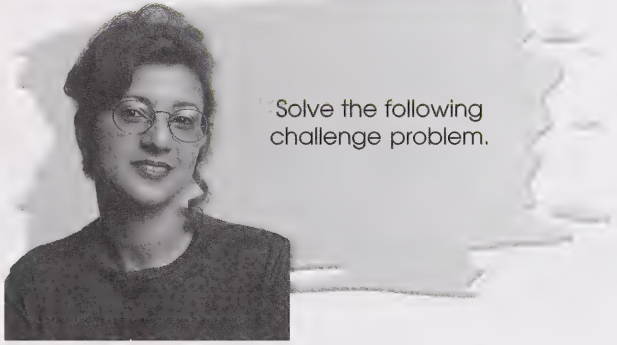
14. Factor the following binomials. Take out the GCF first.

a. $x^3 - 4x$ **b.** $6 - 24x^2$

c. $3x^2 - 3y^2$ **d.** $5n^2 - 500$

Check your answers by turning to the Appendix, page 137.

Now Try This



15. a. Using algebra tiles, display $x^2 + 4x - 5$ as a rectangle.

b. What are the dimensions of the rectangle?

c. Explain how algebra tiles are used to find the factors of a polynomial, where the tiles do not form a complete rectangle at the start.

Check your answers by turning to the Appendix, page 137.

Looking Back

In this activity, you discovered how to factor trinomials of the form $x^2 + bx + c$ using algebra tiles and paper and pencil. This is a skill you will use frequently in your future mathematics courses.

16. In your journal, copy the following procedure to factor a polynomial of the form $x^2 + bx + c$.

> To factor a polynomial of the form $x^2 + bx + c$, you need to find two numbers whose product is c and whose sum is b.

Now, complete the factorization of $x^2 - 7x + 12$ in your journal. Check your solution using FOIL.

Check your answer by turning to the Appendix, page 138.

Conclusion

In this section, you began the process of learning how to factor polynomials by reviewing what is meant by factors and factoring. You used algebra tiles to factor simple polynomials, and extended this to finding common factors for monomials and polynomials. You took another step further when you examined expressions that had a common binomial factor. Finally, you factored trinomials into two binomial factors.

You should now have a good grasp of the basics of factoring polynomials, which you will expand upon as you move into future mathematics courses.

When working with polynomials, you took note of how to multiply polynomials to help you discover factoring (the reverse procedure of multiplying). This is similar to finding your way in and out of a city. You learn the route by taking note of important details and special clues along the way. After completing this module, do you feel comfortable working your way in and out of the subdivision called polynomials?

Assignment

Turn to Assignment Booklet 4B and complete the assignment for Section 2.

Module Summary

The algebra puzzle is now a couple of pieces closer to completion. By completing this module, you have fitted two more pieces (concepts)—Operations with Polynomials and Factoring—into your algebra puzzle.

You defined polynomials, examined how to use polynomials in practical settings, and explored how to perform operations involving polynomials. You used algebra tiles to model polynomials and determine how to add, subtract, multiply, divide, simplify, and factor polynomials. You then used your modelling experience to determine how to perform these operations using only paper and pencil.

Module 4 has broadened your knowledge of algebra by introducing you to the world of polynomials. It has provided information necessary for you to proceed in finding other pieces to the algebra puzzle in future mathematics courses.

Final Module Assignment

Turn to Assignment Booklet 4B and complete the final module assignment for Module 4.

Appendix

Problem-Solving Strategies
Glossary
Suggested Answers
Credits
Cut-out Learning Aids

Problem-Solving Strategies

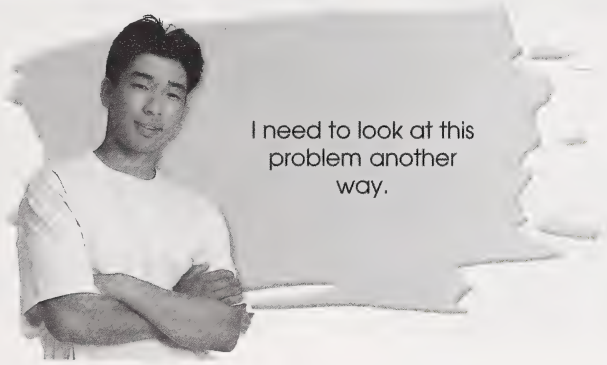
Be sure to study the following problem-solving strategies and use them when you encounter problems in this course.

Whichever method of problem solving you may choose to use, consider the following:

- Show that you understand the problem by showing all the steps needed for finding the answer.
- Draw and neatly label any diagram, graph, or chart that may help you answer the problem.
- Write a concluding sentence that answers the question being asked.

Changing Your Point of View

Sometimes you find that you cannot solve a particular problem because of your "mind set." Perhaps you made an assumption about the problem that is incorrect. If your attempts to solve a problem are not successful, it is often helpful to try to change your point of view.

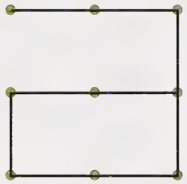


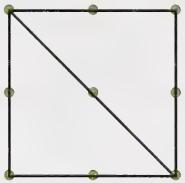
Example

Draw four line segments through the following nine dots without lifting your pencil from the paper and without retracing your path.

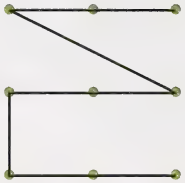
Solution

Step 1: You make several attempts at solving the problem.


5 line segments

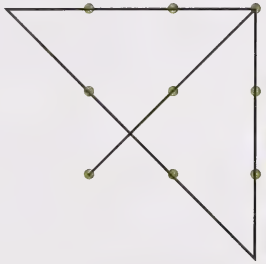

5 line segments

5 line segments


5 line segments

Step 2: You realize you are blocked in your attempts. You have made the assumption that the dots must be connected in a way similar to a child's dot-to-dot picture.

In order to solve the problem, you must change your point of view and realize that the line segments may lie outside the confines of the dots.



4 line segments

Using Objects

Objects can be used to help you solve a problem.

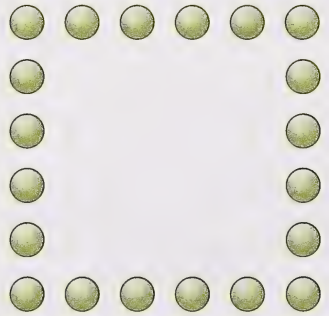
Example

To enclose a square lot with a fence, 20 posts are used. If the posts are placed 3 m apart, what is the length of one side of the lot?

Solution

Step 1: Use objects such as blocks, checkers, or coins to represent the posts.

The advantage of using objects to represent the posts is that you can easily rearrange them.



Step 2: Count the number of spaces between the posts on one side of the square.

There are five spaces between the six posts on one side of the square.

Step 3: Calculate the length of one side of the square.

$$5 \times 3 = 15$$

The length of one side of the lot is 15 m.

Example

Four days before the day after tomorrow was Monday. What day of the week is it today?

Solution

Step 1: Use paper cups to represent the days. The cups could be labelled as shown.

Step 2: Begin at the cup representing the day after tomorrow and count back four days. That day was Monday.

If Monday was the day before yesterday, yesterday was Tuesday, and today is Wednesday.

Using Diagrams

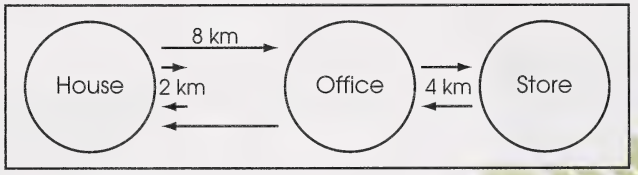
You can use sketches to solve problems.

Example

Chris's office is 8 km from his home. Yesterday morning he drove 2 km before he realized that he had forgotten his briefcase. He returned home to get his briefcase and then drove to his office. At noon, Chris drove 4 km to a store. He then went back to the office for the rest of the afternoon. At the end of the day, he drove straight home. How far did he drive yesterday?

Solution

Step 1: Draw and label a diagram to help you understand the problem.



Step 2: Calculate the distance Chris drove.

$$2+2+8+4+4+8=28$$

Chris drove 28 km.

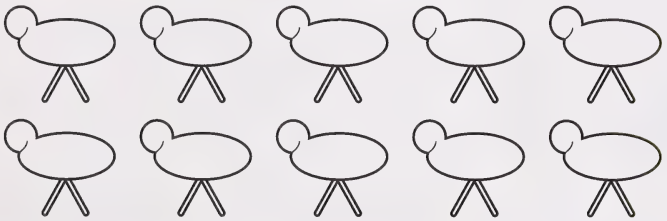
Example

There are 10 animals in a barnyard. Some are chickens and the rest are sheep. Ruth counts 28 legs. How many chickens and how many sheep are there? **Hint:** Chickens have 2 legs and sheep have 4 legs.

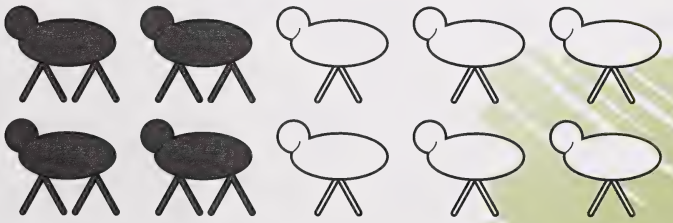
Solution

Draw a diagram to help you solve the problem.

Step 1: Every animal has at least 2 legs. So, draw a diagram of 10 animals, each of which has at least 2 legs.



Step 2: The group of animals in the problem had 28 legs. So, you need to add 8 legs to the drawing. Add the legs in pairs to the sketch of each animal.



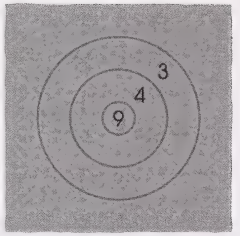
There are 4 sheep and 6 chickens.

Making an Organized List

Some problems require you to list all the possible solutions. It is important to make an organized list so that you do not miss any possibilities.

Example

Julio and his friends like playing a game called Bull's Eye with lawn darts. They draw a target in the dirt and assign values to the three regions of the target.



If Julio throws three darts and all three hit the target, how many different point totals are possible?

Solution

Begin with the highest possible score and list all the possibilities in order.

$9+9+9=27$ ← highest possible score
$9+9+4=22$
$9+9+3=21$
$9+4+4=17$
$9+4+3=16$
$9+3+3=15$
$4+4+4=12$
$4+4+3=11$
$4+3+3=10$
$3+3+3=9$ ← lowest possible score

There are 10 possible point totals.

Example

At an ice-cream parlour there are 3 flavours of ice cream: chocolate, vanilla, and strawberry. How many different double-scoop ice-cream cones are possible? **Hint:** A chocolate-vanilla cone is different from a vanilla-chocolate one. Order makes a difference.

Solution

Draw a tree diagram to help you make an organized list. Use V for vanilla, C for chocolate, and S for strawberry.

Scoop 1	Scoop 2	Result
V	V	(V, V)
	C	(V, C)
	S	(V, S)
C	V	(C, V)
	C	(C, C)
	S	(C, S)
S	V	(S, V)
	C	(S, C)
	S	(S, S)

Nine different double-scoop ice-cream cones are possible.

Using Venn Diagrams

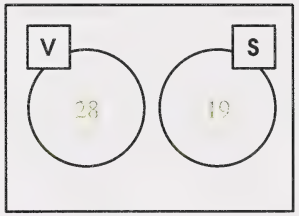
A type of diagram that is helpful in some problems is a Venn diagram. In this special kind of diagram, circles are used to represent groups of people, animals, or objects that have certain characteristics. The positioning of the circles in relation to one another represents relationships among these groups. These diagrams were named after an English mathematician by the name of John Venn (1834–1923).

Example

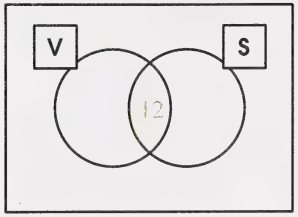
There are 28 girls on the volleyball team and 19 girls on the swim team. If 12 girls belong to both teams, how many girls belong only to the swim team?

Solution

Step 1: Represent the members of the volleyball team with a circle. Represent the members of the swim team with another circle.



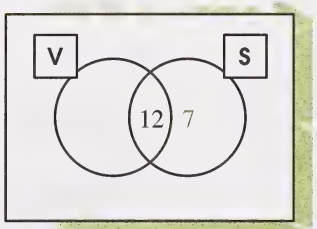
Step 2: Represent the members of both teams by the intersection of the two circles.



Step 3: Now, calculate the number of girls who belong only to the swim team.

$$19 - 12 = 7$$

So, 7 girls belong only to the swim team.

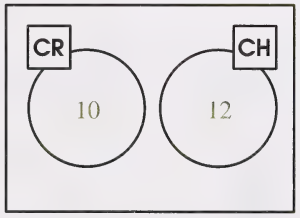


Example

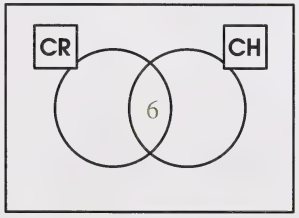
Reilly bought some plates at a garage sale. He discovered that 10 plates were cracked, 12 plates were chipped, 6 plates were both chipped and cracked, and 2 plates were neither chipped nor cracked. How many plates did Reilly buy?

Solution

Step 1: The problem states that 10 plates were cracked (CR) and 12 plates were chipped (CH). Represent the cracked plates with a circle. Represent the chipped plates with a second circle.

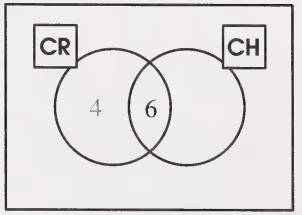


Step 2: The problem states that 6 plates were both chipped and cracked. Represent the cracked and chipped plates with the intersection of the two circles.



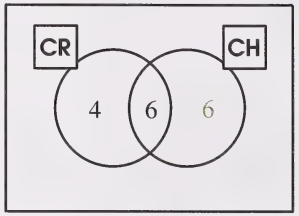
Step 3: Calculate the number of plates that were only cracked.

$$10 - 6 = 4$$

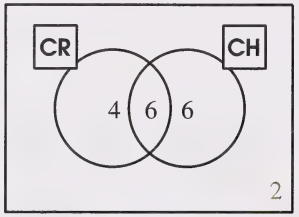


Step 4: Calculate the number of plates that were only chipped.

$$12 - 6 = 6$$



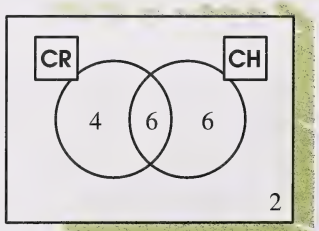
Step 5: The problem states that 2 plates were neither chipped nor cracked. Represent the undamaged plates with the region outside the circles.



Step 6: Calculate the total number of plates that were bought.

$$4 + 6 + 6 + 2 = 18$$

So, 18 plates were bought.



Example

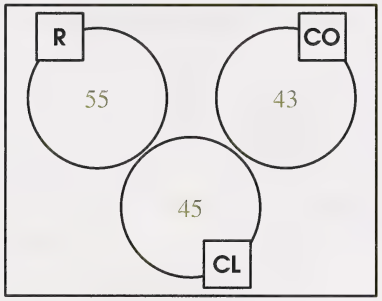
In a survey of 100 people, the following information was collected:

- 43 people like country music
- 55 like rock music
- 45 like classical music
- 8 people like all three forms
- 15 people like both country and classical music
- 25 like both rock and country music
- 20 like both rock and classical music

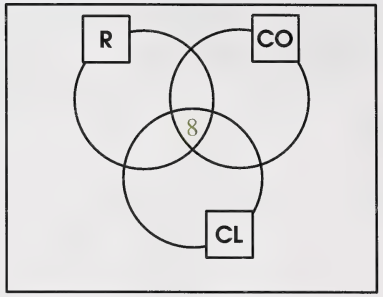
How many people do not like any of these forms of music?

Solution

Step 1: The problem states that 55 people like rock music (R), 43 like country music (CO), and 45 like classical music (CL). Use three circles to represent these preferences.

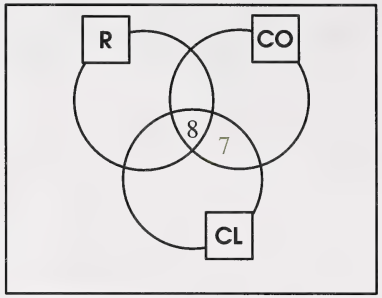


Step 2: The problem states that 8 people like all three forms. The intersection of the three circles shows the people who like all three forms.



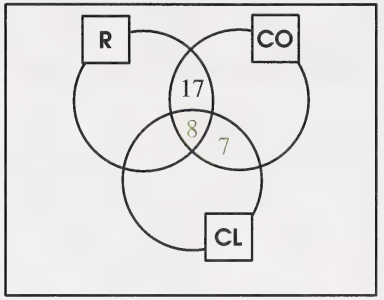
Step 3: The problem states that 15 people like both country and classical music. Calculate the number of people who like country and classical music, but not rock music.

$15 - 8 = 7$



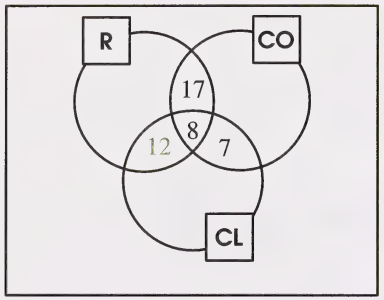
Step 4: The problem states that 25 people like both rock and country music. Calculate the number of people who like both country and rock music, but not classical music.

$25 - 8 = 17$



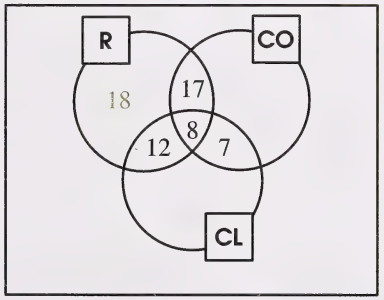
Step 5: The problem states that 20 people like both rock and classical music. Calculate the number of people who like both rock and classical music, but not country music.

$$20 - 8 = 12$$



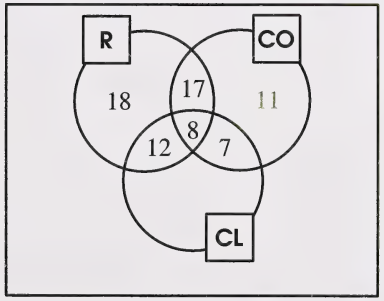
Step 6: Now that you know the number of people who like rock music and any other form of music, calculate the number of people who only like rock.

$$55 - (17 + 8 + 12) = 18$$



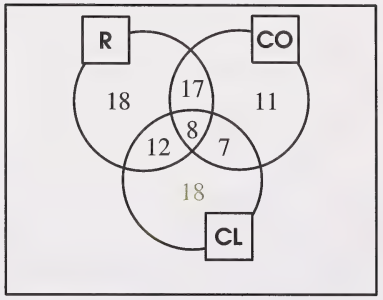
Step 7: Now that you know the number of people who like country music and any other form of music, calculate the number of people who only like country music.

$$43 - (17 + 8 + 7) = 11$$



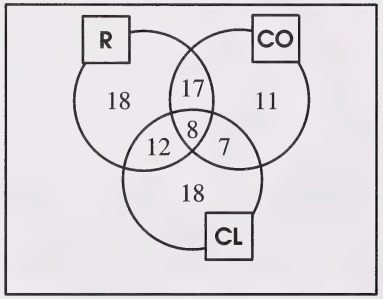
Step 8: Now that you know the number of people who like classical music and any other form of music, calculate the number of people who only like classical music.

$$45 - (12 + 8 + 7) = 18$$



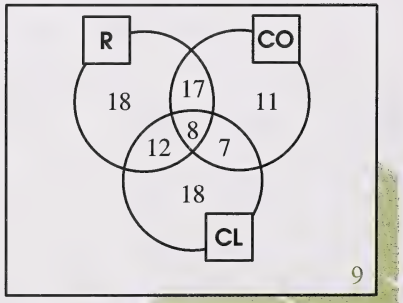
Step 9: Now, you can calculate the total number of people who like rock, country, or classical music.

$$18 + 17 + 8 + 12 + 11 + 7 + 18 = 91$$



Step 10: The problem states that 100 people were surveyed. So, you can calculate the number of people who do not like any of these forms of music.

$$100 - 91 = 9$$



Of the 100 people surveyed, 9 people do not like any of these forms of music.

Making a Table

You can use a table to organize information.

Example

Hannah works at her mother's store after school. She earns 20¢ for each customer that she assists and 35¢ for each bag that she packs. One day she did 19 jobs and earned $4.85. How many bags did she pack and how many customers did she assist that day?

Solution

Write down all possible combinations of 19 jobs until you find a combination that totals $4.85. Use a table to organize the information.

In the first column, put 1 bag packed and 18 customers assisted. Calculate the money received for that.

$$\begin{aligned} 1 \times 35¢ &= \$\ .35 \\ 18 \times 20¢ &= \$3.60 \\ \text{Total} \quad & \$3.95 \end{aligned}$$

Then complete the table for other combinations.

Packing	1	2	3	4	5	6	7
Assisting	18	17	16	15	14	13	12
Earned	$3.95	$4.10	$4.25	$4.40	$4.55	$4.70	$4.85

Hannah packed 7 bags and assisted 12 customers that day.

Guessing, Checking, and Revising

Many mathematical problems can be solved by systematic trial or guessing, checking, and revising.

Example

Shelley opened her math textbook and noticed that the product of the numbers of the two open pages was 2970. To what pages was the book opened? **Hint:** The left-hand page number of a book is even; the right-hand page number is odd.

Solution

Method 1: Using Paper and Pencil

Step 1: Note the conditions that need to be met.

- The left-hand page number must be even.
- The right-hand page number is one more than the left-hand page number.
- The product of the numbers must be 2970.

Step 2: Make a guess; check your guess; and revise if necessary.

It is helpful to organize your guesses in a table.

Left-Hand Page Number	50	52	54
Right-Hand Page Number	51	53	55
Product of Page Numbers	2550	2756	2970

correct guess

So, the book was opened to pages 54 and 55.

Method 2: Using a Computer

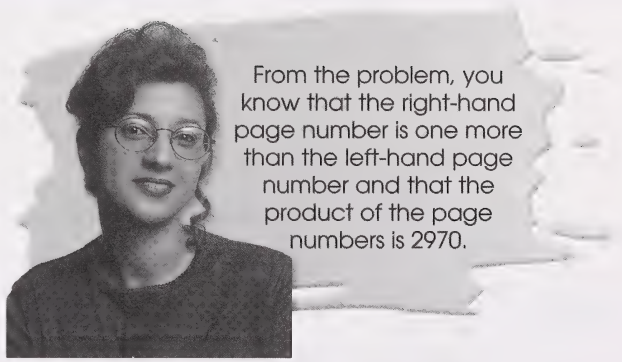
Use a computer and a spreadsheet program to help you solve the problem.

Step 1: Label the cells in the spreadsheet. **Hint:** The cell address at the top of the spreadsheet shows which cell is active. There is also a border around the active cell. You don't type directly in this cell; instead, you use the entry bar at the top of the spreadsheet.

A5 ✗ ✓ Product of page numbers

	A	B
1	**Book Problem**	
2		
3	Left-hand page number	
4	Right-hand page number	
5	Product of page numbers	
6		

Step 2: Enter either a number or a formula for each of the cells.

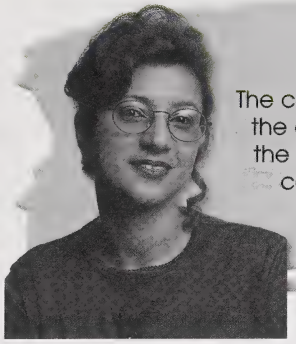

From the problem, you know that the right-hand page number is one more than the left-hand page number and that the product of the page numbers is 2970.

Here is what you enter:

- Begin with cell B3 and enter a guess of 50.
- Next, enter "=B3+1" in cell B4. This means the number in the B4 cell will be one more than the number in the B3 cell.
- Finally, enter "=B3*B4" in cell B5. This means that the number in the B5 cell will be the product of the numbers in the B3 and B4 cells.

B5 ✗ ✓ =B3*B4

	A	B
1	**Book Problem**	
2		
3	Left-hand page number	50
4	Right-hand page number	51
5	Product of page numbers	2550
6		



The computer performs the calculations and the results appear in cells B4 and B5.

Step 3: Check your initial guess, and revise it if necessary. The product is 2970, not 2550; therefore, revise your guess.

Delete the number in the B3 cell and enter a guess of 54.

B3 ✗ ✓ 54

	A	B
1	**Book Problem**	
2		
3	Left-hand page number	54
4	Right-hand page number	55
5	Product of page numbers	2970
6		

The computer performs the calculations and different results appear in cells B4 and B5.

Step 4: Check your initial guess, and revise it if necessary. The product is 2970.

So, the book was opened to pages 54 and 55.

Acting Out a Problem

For some problems, you may find it helpful to physically act out the problem situation.

Example

How many cards does each boy have?

Solution

Step 1: Ask yourself these questions:

- What would happen if Jon gave Matt a card? The first condition to be met is that they must have an equal number of cards after the exchange.

- What would happen if Matt gave Jon a card? The second condition to be met is that Jon will have twice as many cards as Matt after the exchange.

Step 2: Make a guess and act out the problem. If your guess does not fit the conditions, revise your guess and repeat the process.

It is helpful to organize your guesses in a table.

Matt's cards	1	2	3	4	5
Jon's cards	3	4	5	6	7

correct guess

So, Matt has 5 cards and Jon has 7 cards.

Note

If Jon gives Matt a card, each boy will have an equal number of cards after the exchange, 6.

If Matt gives Jon a card, Jon will have twice as many cards as Matt after the exchange. (Jon will have 8 cards and Matt will have 4 cards.)

Working Backwards

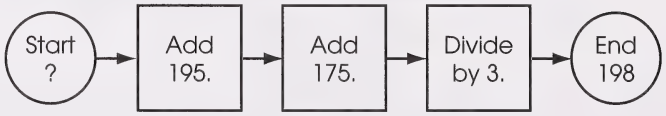
In mathematics, you are usually given information and then asked to find the answer. Sometimes, however, you are given the answer and then asked to find a piece of information. In cases like these, you need to work backwards.

Example

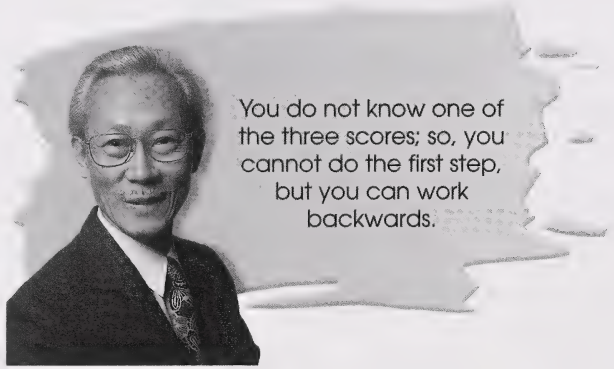
Ross plays in a bowling league. His average in the league is 198. Today Ross scored 195 and 175 in his first two games. What must he score in his third game to maintain his average?

Solution

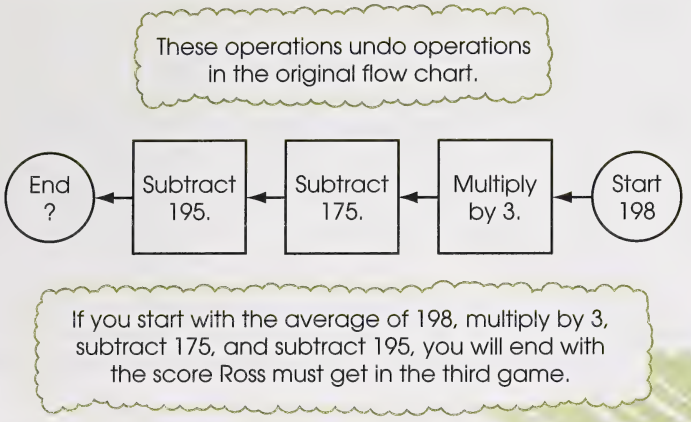
Step 1: Consider how the problem would be done if you worked forwards. This flow chart shows the sequence of the steps.



You start with the score of the third game, add each of the scores of the other two games, divide by 3, and end with the average of 198.



Step 2: Use a reverse flow chart to work backwards.

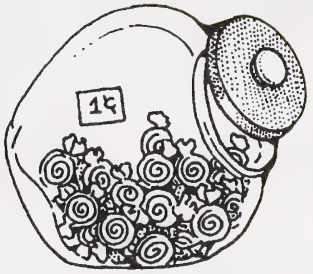

These operations undo operations in the original flow chart.
End ?
Subtract 195.
Subtract 175.
Multiply by 3.
Start 198
If you start with the average of 198, multiply by 3, subtract 175, and subtract 195, you will end with the score Ross must get in the third game.

Ross must score 224 in the last game to maintain his average.

Note: You can check to see if 224 is the correct third score by using the flow chart in Step 1.

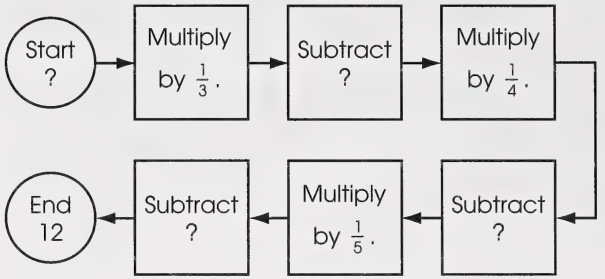
Example

Ruth has a jar of candies. She gives Raschid one-third of her candies. She then gives Steven one-fourth of the remaining candies. Finally, she gives one-fifth of the candies that she has left to Rachel. If Ruth has 12 candies left at the end, how many did she have at the beginning?



Solution

Step 1: Use a flow chart to show the order of operations.

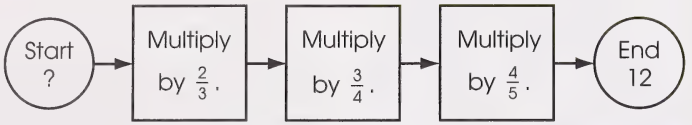


This flow chart is **not** helpful because you do not know how much to subtract in the second, fourth, and sixth boxes.

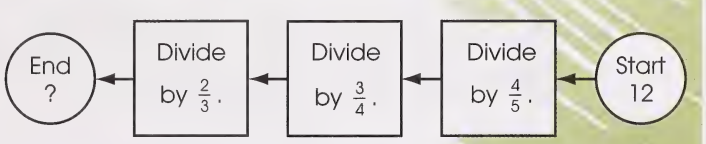
Knowing the following relationships will help you draw a more complete flow chart.

- Giving away one-third of the candies is the same as multiplying the number of candies by two-thirds. $1-\frac{1}{3}=\frac{2}{3}$
- Giving away one-fourth of the remaining candies is the same as multiplying by three-fourths. $1-\frac{1}{4}=\frac{3}{4}$
- Giving away one-fifth of the remaining candies is the same as multiplying by four-fifths. $1-\frac{1}{5}=\frac{4}{5}$

The following flow chart is more useful.



Step 2: Use a reverse flow chart to work backwards.



Ruth had 30 candies at the beginning.

Simplifying a Problem

Are you sometimes confused about which operations to perform in a problem because the numbers in the problem are very large or because the problem seems very complicated? There are strategies you can use to simplify a problem.

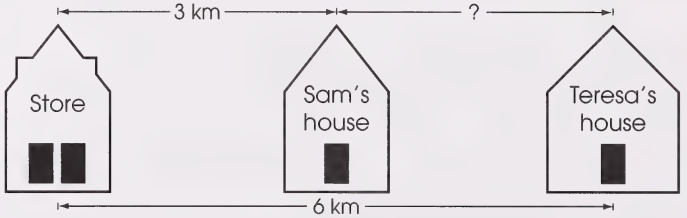
Example

The average distance from the Sun to Mars is 228 000 000 km, and the average distance from the Sun to Earth is 150 000 000 km. What is the average distance between Earth and Mars?

Solution

Step 1: If you find the large numbers overwhelming, make up a related but simpler problem.

Sam and Teresa both live east of the store. Sam lives 3 km from the store and Teresa lives 6 km from the store. How far is it from Sam's house to Teresa's house?



$6 - 3 = 3$

It is 3 km from Sam's house to Teresa's house.

Step 2: Once you understand the related but simpler problem, you can solve the original problem.

$$228\,000\,000 - 150\,000\,000 = 78\,000\,000$$

The average distance from Earth to Mars is 78 000 000 km.

Finding and Applying a Pattern

To solve some problems, it is often helpful to use a related but simpler problem to discover a pattern.

Example

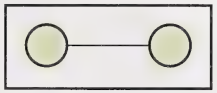
There are 20 students in a room. If every student shakes hands with every other student in the room, how many handshakes are exchanged?

Solution

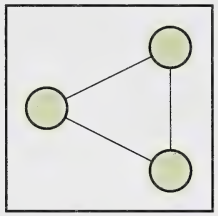
Step 1: Try solving the problem with 1, 2, 3, 4, or 5 students.

- If there is 1 student, no handshakes can be exchanged (as illustrated in the following diagram).

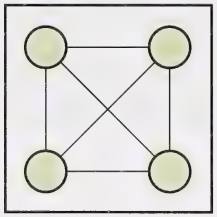
- If there are 2 students, 1 handshake can be exchanged (as illustrated in the following diagram).



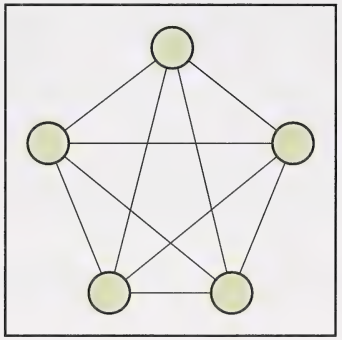
- If there are 3 students, 3 handshakes can be exchanged (as illustrated in the following diagram).



- If there are 4 students, 6 handshakes can be exchanged (as illustrated in the following diagram).



- If there are 5 students, 10 handshakes can be exchanged (as illustrated in the following diagram).

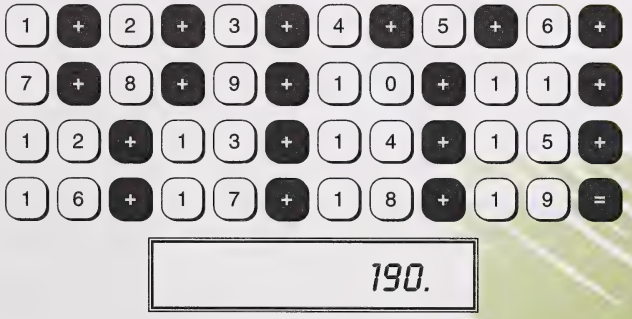


Step 2: Look for a pattern.

People	Handshakes	Pattern
1	0	
2	1	+1
3	3	+2
4	6	+3
5	10	+4

Step 3: Apply the pattern.

A calculator is helpful for applying the pattern. Go up to 19, because each student will shake the hand of 19 others.

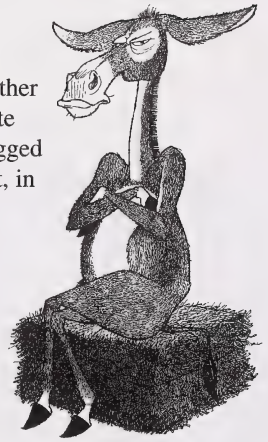


There are a total of 190 handshakes exchanged.

Using Elimination

Detectives are skilful at eliminating clues and solving mysteries. You can become skilful at using elimination to solve problems too.

Example



Five animals had a race over a short distance. Neither the donkey nor the coyote beat the lion. The coyote beat the rabbit, but not the ostrich. The donkey lagged behind the rabbit. If the ostrich was not the fastest, in what order did the animals finish the race?

Solution

Step 1: Decide what clues are given. The clues indicate the following:

- The donkey and the coyote did not finish first because they did not beat the lion.
- The rabbit did not finish first because the coyote beat it.
- The ostrich was faster than the coyote.
- The rabbit was faster than the donkey.
- The ostrich did not finish first because it was not the fastest.

The clues can be shown on a table.

	Donkey	Coyote	Rabbit	Ostrich	Lion
First	✗	✗	✗	✗	
Second					
Third					
Fourth					
Fifth					

Step 2: Use elimination and reconsider the clues.

By elimination, it is clear that the lion was first. Now, reconsider the clues:

- The rabbit did not finish second because the coyote beat it.
- The coyote did not finish second because the ostrich was faster.
- The donkey did not finish second because it lagged behind the rabbit.

The additional clues can be shown in the table.

	Donkey	Coyote	Rabbit	Ostrich	Lion
First	✗	✗	✗	✗	✓
Second	✗	✗	✗		✗
Third					✗
Fourth					✗
Fifth					✗

Step 3: Use elimination and reconsider the clues.

By elimination, it is clear that the ostrich finished second. Now, reconsider the clues:

- The rabbit did not finish third because the coyote beat it.
- The donkey did not finish third because it lagged behind the rabbit.

The additional clues can be shown in the table.

	Donkey	Coyote	Rabbit	Ostrich	Lion
First	✗	✗	✗	✗	✓
Second	✗	✗	✗	✓	✗
Third	✗		✗	✗	✗
Fourth				✗	✗
Fifth				✗	✗

Step 4: Use elimination and reconsider the clues.

By elimination, it is clear that the coyote finished third. Now, reconsider the clues.

Because the donkey lagged behind the rabbit, the rabbit was fourth and the donkey was fifth.

	Donkey	Coyote	Rabbit	Ostrich	Lion
First	✗	✗	✗	✗	✓
Second	✗	✗	✗	✓	✗
Third	✗	✓	✗	✗	✗
Fourth	✗	✗	✓	✗	✗
Fifth	✓	✗	✗	✗	✗

So, the lion was first, the ostrich was second, the coyote was third, the rabbit was fourth, and the donkey was last.

Using Truth Tables

A strategy you can use to solve some logic problems is making truth tables. Truth tables are similar to elimination tables.

Example

Detective Beagle is investigating a robbery in the forest. Charlie Chipmunk and Sammy Squirrel each deny being the robber. Harold Hare blames Charlie Chipmunk. Detective Beagle knows two of the animals are lying and one is telling the truth. Who is the robber?

Solution

Step 1: Assume that Charlie Chipmunk was the robber.

- Charlie Chipmunk said he didn't do it. If Charlie Chipmunk was the robber, this is a false statement. Make a truth table and put **F** under Charlie's name.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F		

- Sammy Squirrel denied doing it. If Charlie Chipmunk was the robber, this is a true statement. Put **T** under Sammy's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	**T**	

- Harold Hare blamed Charlie Chipmunk. If Charlie Chipmunk was the robber, this is a true statement. Put **T** under Harold's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	**T**

Detective Beagle knows two animals are lying. So, Charlie is not the robber.

Step 2: Assume that Sammy Squirrel was the robber.

- Charlie Chipmunk denied doing it. If Sammy Squirrel was the robber, this is a true statement. Put **T** under Charlie's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	T
T		

- Sammy Squirrel denied doing it. If Sammy Squirrel was the robber, this is a false statement. Put **F** under Sammy's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	T
T	**F**	

- Harold Hare blamed Charlie Chipmunk. If Sammy Squirrel was the robber, this is a false statement. Put **F** under Harold's name in the truth table.

Charlie Chipmunk	Sammy Squirrel	Harold Hare
F	T	T
T	F	**F**

Detective Beagle knows two of the animals are lying and one is telling the truth.

So, Sammy Squirrel was the robber.

Using an Equation

A strategy you can use to solve some problems is to write and solve an equation.

Example

Twelve less than a number is -3. Find the number.

Solution

Step 1: Let the unknown be equal to a variable. Let the number be n.

Step 2: Write an equation.

$$n - 12 = -3$$

Twelve less than a number is -3.

Step 3: Solve the equation.

$$\begin{array}{r} n - 12 = -3 \\ +12 \quad +12 \\ \hline n \quad = \quad 9 \end{array}$$

The number is 9.

Example

The second of two numbers is 7 times the first. The sum of the two numbers is 32. Find the numbers.

Solution

Step 1: Write statements for the unknowns. Use only one variable.

Let n be the first number and let $7n$ be the second number.

The second of two numbers is 7 times the first.

Step 2: Write an equation.

$$n + 7n = 32$$

The sum of the two numbers is 32.

Step 3: Solve the equation.

$$\begin{aligned} n + 7n &= 32 \\ 8n &= 32 \\ \frac{8n}{8} &= \frac{32}{8} \\ n &= 4 \end{aligned}$$

The first number is 4.

Step 4: Find the second number if the first number is 4.

$$\begin{aligned} 7n &= 7 \times 4 \\ &= 28 \end{aligned}$$

The second number is 28.

So, the numbers are 4 and 28.

Example

Yvonne has 20 more nickels than dimes in her piggy bank. If the total value of the nickels and dimes is $8.50, how many coins of each type does she have?

Solution

Step 1: Write statements for the unknowns. Use only one variable.

Let the number of dimes be n.

The value of the nickels, the number of dimes, and the value of the dimes must also be represented by algebraic expressions.

A table can be used to organize the information.

Type of Coin	Number of Coins	Value in Cents
Dimes	n	$10n$
Nickels	$n + 20$	$5(n + 20)$
Total		850

The value of the dimes is 10 times the number of dimes.

The value of the nickels is 5 times the number of nickels.

Yvonne has 20 more nickels than dimes.

This is given.

Step 2: Write an equation.

$$5(n+20)+10n=850$$

Step 3: Solve the equation.

$$\begin{aligned} 5(n+20)+10n &= 850 \\ 5n+100+10n &= 850 \\ 15n+100 &= 850 \\ -100 \quad -100 & \\ \hline 15n \quad &= 750 \\ \frac{15n}{15} &= \frac{750}{15} \\ n &= 50 \end{aligned}$$

Yvonne has 50 dimes in her piggy bank.

Step 4: Find the number of nickels if the number of dimes is 50.

$$\begin{aligned} n+20 &= 50+20 \\ &= 70 \end{aligned}$$

Yvonne has 70 nickels in her piggy bank.

So, Yvonne has 50 dimes and 70 nickels in her piggy bank.

Glossary

ascending order: in order from smallest to largest

binomial: a polynomial with two terms

common factor: a factor that is common to two or more numbers or terms

constant: a term that does not contain a variable

In the polynomial $x^2 + 5x + 6$, 6 is the constant.

degree of monomial: the sum of the exponents of all of the variables in a monomial

The monomial $4xy^2$ is of degree 3.

degree of a polynomial: the degree of the term that has the largest sum of exponents

The polynomial $3xy + 4x^2y^2 + 9x - 4$ is of degree 4.

degree of a term: the sum of the exponents of all the variables in a term

descending order: in order from largest to smallest

distributive property: a rule for expanding an expression within brackets by multiplying each term inside the brackets by the term outside the brackets

$4(7+6) = (4\times 7) + (4\times 6)$ $\qquad$ $3(n-2) = 3n - 6$

expanding: removing the brackets and simplifying an expression

$3(x^2 + 2x - 3) = 3x^2 + 6x - 9$

factor: to write a number or polynomial as a product

$8 = 2\times 2\times 2$ $\qquad$ $4m^2n = (4)(m)(m)(n)$

FOIL: an acronym for the method of multiplying two binomials

First terms, **O**utside terms, **I**nside terms, **L**ast terms

greatest common factor (GCF): the largest factor that a group of numbers or terms has in common

The GCF of $12x^3y$, $16x^2y^2$, and $8xyz$ is $4xy$.

like terms: terms with identical literal coefficients

$4x^2y$, $-6x^2y$, and x^2y are like terms.

literal coefficient: a non-numerical factor (or factors) of an algebraic terms

mathematical expression: a combination of numerals, variables, and other mathematical symbols

$4x$ $\qquad$ $3n^2 + 4n - 1$ $\qquad$ $\dfrac{x+3}{x-4}$

monomial: a polynomial with one term

numerical coefficient: the numerical factor of an algebraic term

polynomial: an algebraic expression in which there are no variables in a denominator, no variables under a root sign, and no variables with fractional exponents

$x^2 + 5x,\ 4x^3y$

simplest form: a fraction written so the numerator and denominator have no common factors

$\frac{10}{12} = \frac{5}{6}$ $\qquad$ $\frac{4xy}{6xz} = \frac{2y}{3z}$

term: a mathematical expression consisting of the product of only numbers and/or variables with whole-number exponents

$4a^2b,\ 15n,\ -5mn$

trinomial: a polynomial with three terms

$5x^2 + 2x + 1$

variable: a letter that represents a number

zero principle: the principle that two opposites cancel one another or result in zero

Suggested Answers

Section 1: Activity 1

1. a. yes

b. yes

c. no
There is a variable under the radical sign.

d. yes

e. no
There is a variable in each of the denominators.

f. no
There is a variable in the denominator.

2. a. The numerical coefficient is 7.
The literal coefficient is ab.
The variables are a and b.

b. The numerical coefficient is -8.
The literal coefficient is y^3.
The variable is y.

c. The numerical coefficient is 4.
The literal coefficient is xyz.
The variables are x, y, and z.

d. The numerical coefficient is -6.
The literal coefficient is z.
The variable is z.

e. The numerical coefficient is 2.
The literal coefficient is x^2y^3.
The variables are x and y.

f. The numerical coefficient is $\frac{1}{2}$.
The literal coefficient is ab.
The variables are a and b.

3. The terms are grouped as follows:

- m^2n, $2m^2n$, $\frac{m^2n}{2}$, and $-8m^2n$ are like terms.
- $-5a$, $4a$, and $\frac{16a}{3}$ are like terms.
- $2mn$ and mn are like terms.
- -9 and 6 are like terms.
- $-3a^2b$ and $-12a^2b$ are like terms.
- $-2ab$ and $-5ab$ are like terms.

4. The expression $6a^{-2}+\frac{4}{a}-2$ is not a polynomial according to the definition; it has a negative exponent and a variable in the denominator. The expression $2a+3b+4c+5d$ is a polynomial, but it has four terms.

5. **a.** monomial **b.** monomial **c.** binomial
d. trinomial **e.** trinomial **f.** trinomial
g. binomial **h.** monomial **i.** trinomial

6. **a.** Ascending order: $8+7a+3a^2$
Descending order: $3a^2+7a+8$

b. Ascending order: $6-4b+8b^2+2b^3$
Descending order: $2b^3+8b^2-4b+6$

c. Ascending order: $16+2c$
Descending order: $2c+16$

d. Ascending order: $-4-2mn+6m^2n+4m^3n^2$
Descending order: $4m^3n^2+6m^2n-2mn-4$

e. Ascending order: $9+3xy-x^2y^2+3x^3y^2$
Descending order: $3x^3y^2-x^2y^2+3xy+9$

7. **a.** 3 **b.** 3 **c.** 3
d. 0 **e.** 6 **f.** 4

8. **a.** 2 **b.** 2 **c.** 4
d. 1 **e.** 5 **f.** 3

9.

Polynomial	Number of Terms (by name)	Degree
$8a^2+6a-2$	trinomial	2
$7b^3-3b$	binomial	3
16	monomial	0
$2d$	monomial	1
$2x^2y^3-4xy+7y$	trinomial	5
$10m^2n^4+6mn$	binomial	6
$4y^4+3y^3-2y^2+y-1$	polynomial	4

10. Answers will vary. Some examples have been provided.

a. $5xy^3,\ 2x^2y^2,\ -3x^3y$

b. $4a^2-2a+7,\ a^2+5a-2$

c. $7a+8,\ a+4,\ 2a-9$

d. $6a^5+4a^3+2a^2+a-8,\ 8a^5+2a^4-6a+2$

e. $x^5y-2x^4y^2+3xy,\ 7x^2y^4+2xy-7$

Now Try This

11. Here are four methods of solving this problem.

Method 1: Guessing, Checking, and Revising

If at first you didn't realize that the number of chocolates had to be a multiple of six, you should have after your first guess.

Try 30.

$\frac{1}{6}\times 30=5,\ 30-5=25$

$\frac{1}{5}\times 25=5,\ 25-5=20$

$\frac{1}{4}\times 20=5,\ 20-5=15$

$\frac{1}{3}\times 15=5,\ 15-5=10$

$\frac{1}{2}\times 10=5,\ 10-5=5$

This is too many.

Try 24.

$\frac{1}{6}\times 24=4,\ 24-4=20$

$\frac{1}{5}\times 20=4,\ 20-4=16$

$\frac{1}{4}\times 16=4,\ 16-4=12$

$\frac{1}{3}\times 12=4,\ 12-4=8$

$\frac{1}{2}\times 8=4,\ 8-4=4$

Therefore, there were 24 chocolates in the box originally.

Method 2: Working Backwards

Because 4 is half of 8, there were 8 chocolates left before Anthony started eating them.

8 is $\frac{2}{3}$ of what? (Janine had eaten $\frac{1}{3}$.)

$$\overset{4}{\cancel{8}} \times \frac{3}{\cancel{2}_{1}} = 12$$

There were 12 chocolates left when Janine saw the box.

12 is $\frac{3}{4}$ of what? (Gary had eaten $\frac{1}{4}$.)

$$\overset{4}{\cancel{12}} \times \frac{4}{\cancel{3}_{1}} = 16$$

There were 16 chocolates left before Gary got to the box.

16 is $\frac{4}{5}$ of what? (Irene had eaten $\frac{1}{5}$.)

$$\overset{4}{\cancel{16}} \times \frac{5}{\cancel{4}_{1}} = 20$$

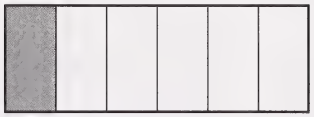
There were 20 chocolates left when Irene got the box.

20 is $\frac{5}{6}$ of what? (Mr. Wilkinson had eaten $\frac{1}{6}$.)

$$\overset{4}{\cancel{20}} \times \frac{6}{\cancel{5}_{1}} = 24$$

Therefore, there were 24 chocolates in the box originally.

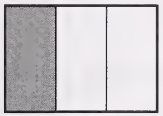
Method 3: Using Diagrams



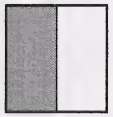
Mr. Wilkinson ate $\frac{1}{6}$ of the chocolates.

Irene ate $\frac{1}{5}$ of the remaining chocolates.

Gary ate $\frac{1}{4}$ of the remaining chocolates.



Janine ate $\frac{1}{3}$ of the remaining chocolates.



Anthony ate $\frac{1}{2}$ of the remaining chocolates.

There are 4 chocolates left.

Therefore, there were $4 \times 6 = 24$ chocolates originally.

Method 4: Using Algebra

Let c be the number of chocolates in the box.

$$c - \frac{1}{6}c = \frac{5}{6}c \quad \text{(number left by Mr. Wilkinson)}$$

$$\frac{5}{6}c - \frac{1}{\cancel{5}}\left(\frac{\cancel{5}}{6}c\right) = \frac{5}{6}c - \frac{1}{6}c$$

$$= \frac{4}{6}c \quad \text{(number left by Irene)}$$

$$\frac{4}{6}c - \frac{1}{\cancel{4}}\left(\frac{\cancel{4}}{6}c\right) = \frac{4}{6}c - \frac{1}{6}c$$

$$= \frac{3}{6}c \quad \text{(number left by Gary)}$$

$$\frac{3}{6}c - \frac{1}{\cancel{3}}\left(\frac{\cancel{3}}{6}c\right) = \frac{3}{6}c - \frac{1}{6}c$$

$$= \frac{2}{6}c \quad \text{(number left by Janine)}$$

$$\frac{2}{6}c - \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{6}c\right) = \frac{2}{6}c - \frac{1}{6}c$$

$$= \frac{1}{6}c \quad \text{(number left by Anthony)}$$

$$\therefore \frac{1}{6}c = 4$$

$$\cancel{6}\left(\frac{1}{\cancel{6}}c\right) = 6(4)$$

$$c = 24$$

There were 24 chocolates originally.

Looking Back

12. The **degree** of the polynomial $5x^2 + 3x - 5$ is two.

An example of a **binomial** is $2x^3 + 6$.

An example of a **monomial** is $4x^2y^3$.

An example of a **trinomial** is $4x^2y + 6x + 2y$.

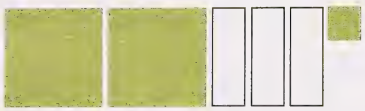
Section 1: Activity 2

1. a. The length of an x-tile and the length of a side of an x^2-tile are the same.

b. The length of a side of a 1-tile and the width of an x-tile are the same.

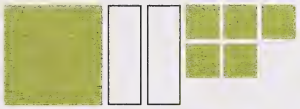
2. a. $2n + 5$ **b.** -4 **c.** $x^2 - 3x + 2$

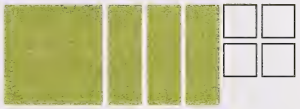
d. $-2m^2 + m - 2$ **e.** $3y^2 + 3y + 4$

3. a. 

b.

c.

d. 

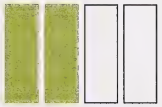
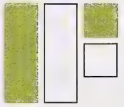
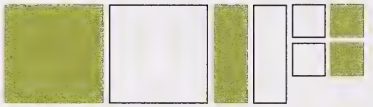
e. 

4. a. $-n^2 - n$ b. $2m + 4$

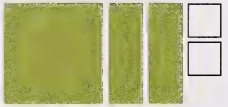
c. $y^2 - y - 2$ d. $-2a^2 + 2a + 3$

Note: Other variables may be used.

5. Answers may vary. Sample models are given.

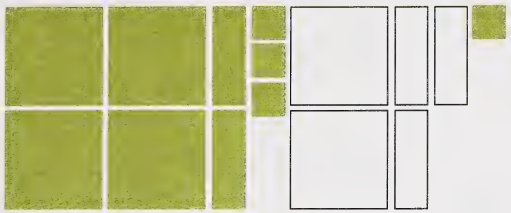
-
- 
- 
- 

6. a. b.

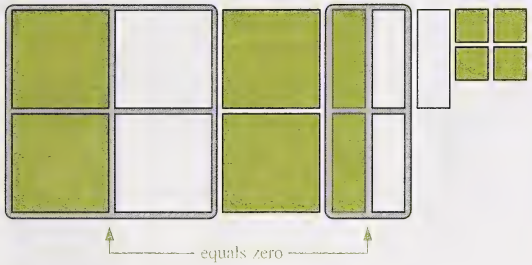
c. d. 

e. f.

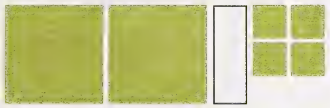
7. a. Model the expression.



Simplify by removing any zero pairs.

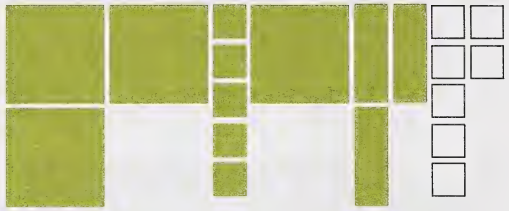


This is the result.

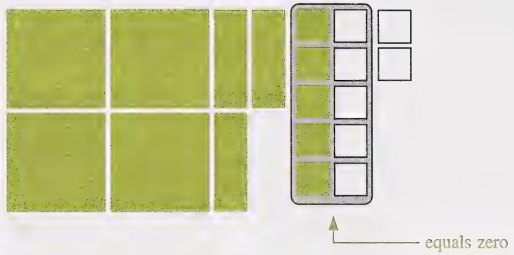


$\therefore\ 4a^2+2a+3-2a^2-3a+1=2a^2-a+4$

b. Model the expression.



Simplify by removing any zero pairs.

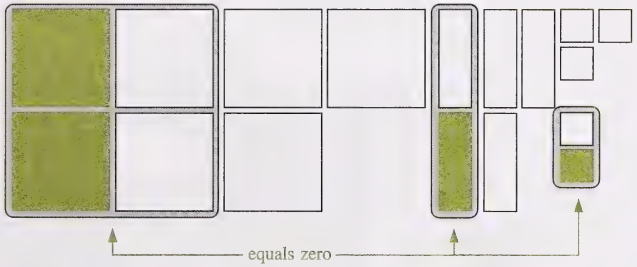


This is the result.

$\therefore\ 3b^2+5+b^2+3b-7=4b^2+3b-2$

c. Model the expression.

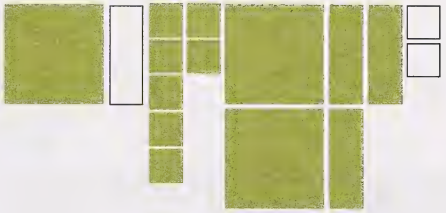
Simplify by removing any zero pairs.



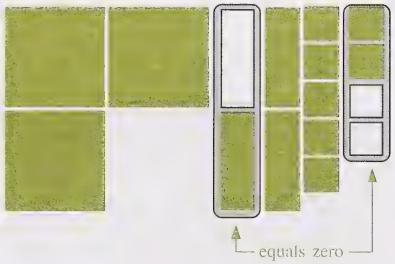
This is the result.

$\therefore\ 2c^2-4x+1-5c^2+c-4=-3c^2-3c-3$

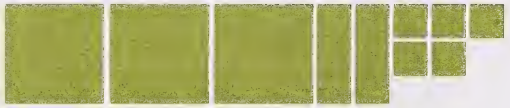
d. Model the expression.



Simplify by removing any zero pairs.



This is the result.



$\therefore\ d^2-d+7+2d^2+3d-2=3d^2+2d+5$

8. To simplify, bring the similar shapes together and eliminate the zeros.

9. a. $7m+2m-3m=6m$

b. $8n^2-2n^2+3n^2=9n^2$

c. $8y+7+4y-5=8y+4y+7-5$
$=12y+2$

d. $3x^2+2x+4x^2-5x+1=3x^2+4x^2+2x-5x+1$
$=7x^2-3x+1$

e. $b^2-2b+4-3b^2+3b-1=b^2-3b^2-2b+3b+4-1$
$=-2b^2+b+3$

f. $2m+m-m-5=2m-5$

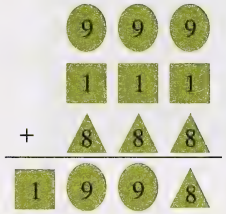
g. $3y^2+4y-2y^2+6y-1=3y^2-2y^2+4y+6y-1$
$=y^2+10y-1$

h. $x^2-y^2+4x+3y-8+x^2-y^2+2x+3$
$=x^2+x^2-y^2-y^2+4x+2x+3y-8+3$
$=2x^2-2y^2+6x+3y-5$

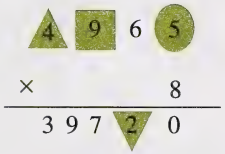
i. $4a^2b+3a^2+7-2b^2+a^2b+4b^2-3$
$=4a^2b+a^2b+3a^2-2b^2+4b^2+7-3$
$=5a^2b+3a^2+2b^2+4$

Now Try This

10. a.

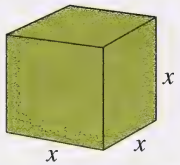


b.



Looking Back

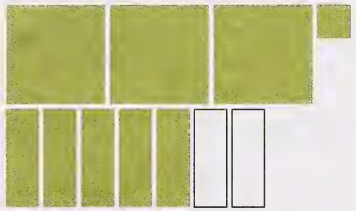
11. a. A cube with length, width, and height of x could be used to represent x^3.



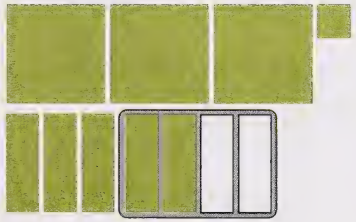
b.

12. The student made the mistake of combining all the numerical coefficients. Only the coefficients of the x-terms should have been combined. This can be shown using algebra tiles.

First, model the initial expression.



Simplify by removing the zero pairs.



This is the result.

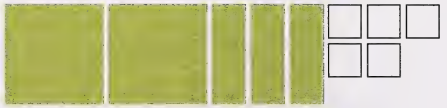
$\therefore\ 3x^2+5x-2x+1=3x^2+3x+1$

Section 1: Activity 3

1. a. Method 1: Using Models

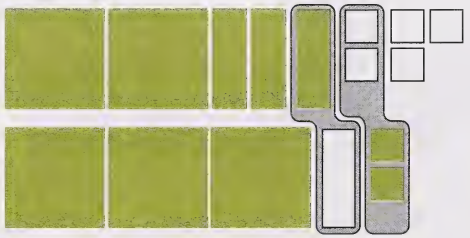
Model each expression.

$2x^2+3x-5$

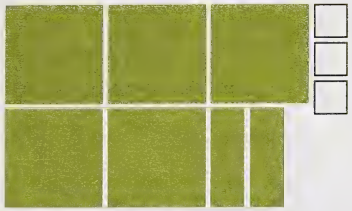


$3x^2-x+2$

Combine the two sets of tiles, and simplify by removing the zero pairs.



This is the result.



$\therefore \left(2x^2+3x-5\right)+\left(3x^2-x+2\right)=5x^2+2x-3$

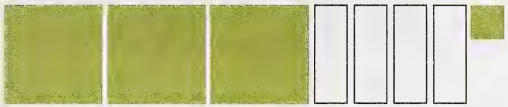
Method 2: Using Paper and Pencil

$$\begin{aligned}\left(2x^2+3x-5\right)+\left(3x^2-x+2\right) &= 2x^2+3x-5+3x^2-x+2\\ &= 2x^2+3x^2+3x-x-5+2\\ &= 5x^2+2x-3\end{aligned}$$

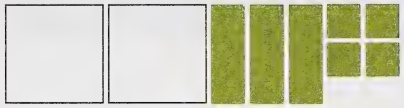
b. Method 1: Using Models

Model each expression.

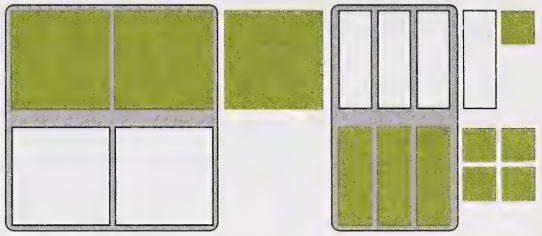
$3y^2-4y+1$



$-2y^2+3y+4$



Combine the two sets of tiles, and simplify by removing the zero pairs.



This is the result.

$\therefore \left(3y^2-4y+1\right)+\left(-2y^2+3y+4\right)=y^2-y+5$

Method 2: Using Paper and Pencil

$$\begin{aligned}\left(3y^2-4y+1\right)+\left(-2y^2+3y+4\right) &= 3y^2-4y+1-2y^2+3y+4\\ &= 3y^2-2y^2-4y+3y+1+4\\ &= y^2-y+5\end{aligned}$$

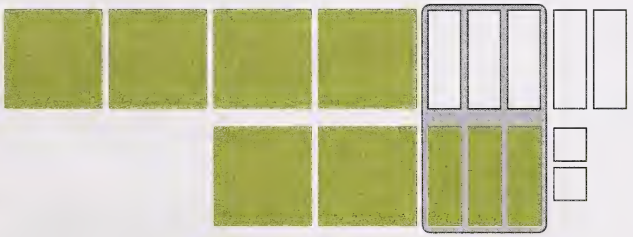
c. Method 1: Using Models

Model each expression.

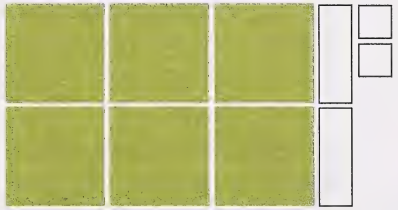
$4a^2 - 5a$

$2a^2 + 3a - 2$

Combine the two sets of tiles, and simplify by removing the zero pairs.



This is the result.



$\therefore \left(4a^2 - 5a\right) + \left(2a^2 + 3a - 2\right) = 6a^2 - 2a - 2$

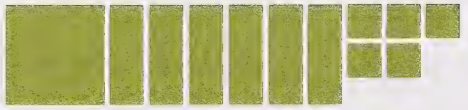
Method 2: Using Paper and Pencil

$$\begin{aligned}\left(4a^2 - 5a\right) + \left(2a^2 + 3a - 2\right) &= 4a^2 - 5a + 2a^2 + 3a - 2 \\ &= 4a^2 + 2a^2 - 5a + 3a - 2 \\ &= 6a^2 - 2a - 2\end{aligned}$$

d. Method 1: Using Models

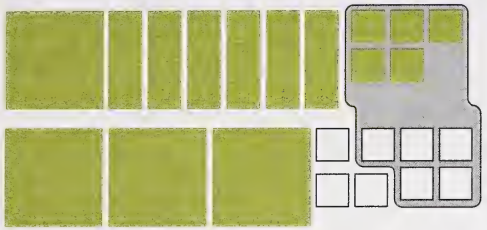
Model each expression.

$y^2 + 6y + 5$

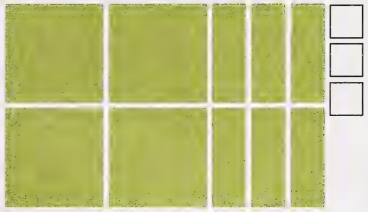


$3y^2 - 8$

Combine the two sets of tiles, and simplify by removing the zero pairs.



This is the result.



$\therefore \left(y^2+6y+5\right)+\left(3y^2-8\right)=4y^2+6y-3$

Method 2: Using Paper and Pencil

$$\begin{aligned}\left(y^2+6y+5\right)+\left(3y^2-8\right) &= y^2+6y+5+3y^2-8\\ &= y^2+3y^2+6y+5-8\\ &= 4y^2+6y-3\end{aligned}$$

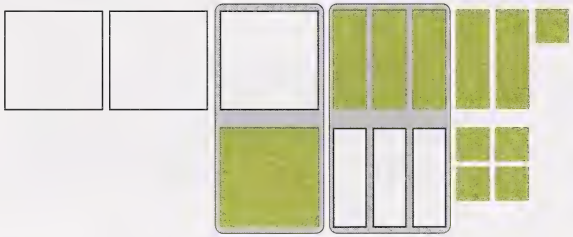
e. Method 1: Using Models

Model each expression.

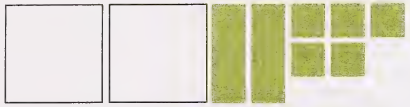
$-3b^2+5b+1$

b^2-3b+4

Combine the two sets of tiles, and simplify by removing the zero pairs.



This is the result.



$\therefore \ (-3b^2+5b+1)+(b^2-3b+4)=-2b^2+2b+5$

Method 2: Using Paper and Pencil

$$\begin{aligned}(-3b^2+5b+1)+(b^2-3b+4) &= -3b^2+5b+1+b^2-3b+4\\ &= -3b^2+b^2+5b-3b+1+4\\ &= -2b^2+2b+5\end{aligned}$$

2. a.
$$\begin{array}{r} 2x^2+7 \\ +\ (3x^2-4) \\ \hline 5x^2+3 \end{array}$$

b.
$$\begin{array}{r} 3a^2+5a+6 \\ +\ (-4a^2+2a-3) \\ \hline -a^2+7a+3 \end{array}$$

c.
$$\begin{array}{r} 8n^2-5n+7 \\ +\ (3n^2-4) \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{r} 8n^2-5n+7 \\ +\ (3n^2\quad -4) \\ \hline 5n^2-5n+3 \end{array}$$

d.
$$\begin{array}{r} 3m^3+2m^2-4m+8 \\ +\ (m^3+7m-6) \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{r} 3m^3+2m^2-4m+8 \\ +\ (m^3\quad\quad +7m-6) \\ \hline 4m^3+2m^2+3m+2 \end{array}$$

3. a. $(2x+3)+(7x)+(5x+1)$

b.
$$\begin{aligned}(2x+3)+(7x)+(5x+1) &= 2x+3+7x+5x+1\\ &= 2x+7x+5x+3+1\\ &= 14x+4\end{aligned}$$

c.
$$\begin{aligned}14x+4 &= 14(3)+4\\ &= 42+4\\ &= 46\end{aligned}$$

The perimeter is 46 cm.

4. a.
$$\begin{aligned}(2x+3)+(3x-2)+(x+8) &= 2x+3+3x-2+x+8\\ &= 2x+3x+x+3-2+8\\ &= 6x+9\end{aligned}$$

b.
$$\begin{aligned}6x+9 &= 6(5)+9\\ &= 30+9\\ &= 39\end{aligned}$$

The perimeter of the plot is 39 m.

c. The plot of land is in the shape of an equilateral triangle, since each side has the same length. You can check this be substituting 5 for x in each of the three parts of the expression.

5. a.
$$\begin{aligned}(4x^2+3x-2)+(x^2-x+5) &= 4x^2+3x-2+x^2-x+5\\ &= 4x^2+x^2+3x-x-2+5\\ &= 5x^2+2x+3\end{aligned}$$

b.

LS	RS
$(4x^2+3x-2)+(x^2-x+5)$	$5x^2+2x+3$
$=[4(2)^2+3(2)-2]+[(2)^2-2+5]$	$=5(2)^2+2(2)+3$
$=[4(4)+3(2)-2]+(4-2+5)$	$=5(4)+2(2)+3$
$=(16+6-2)+7$	$=20+4+3$
$=20+7$	$=27$
$=27$	

LS = RS

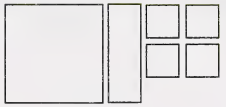
The value for the original expression equals the value for the answer to question 5.a. So, the original expression is equivalent to the answer to question 5.a.

6. a. Method 1: Using Models

Model each expression using the opposite value of each tile in the second polynomial.

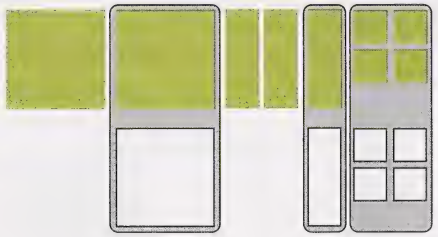
$2x^2+3x+4$

$-x^2-x-4$

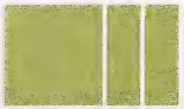


The opposite of x^2+x+4 is $-x^2-x-4$.

Combine the two sets of tiles, and simplify by removing the zero pairs.



This is the result.



$\therefore \left(2x^2+3x+4\right)-\left(x^2+x+4\right)=x^2+2x$

Method 2: Using Paper and Pencil

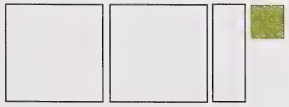
$$\begin{aligned}\left(2x^2+3x+4\right)-\left(x^2+x+4\right)&=\left(2x^2+3x+4\right)+\left(-x^2-x-4\right)\\&=2x^2+3x+4-x^2-x-4\\&=2x^2-x^2+3x-x+4-4\\&=x^2+2x\end{aligned}$$

b. Method 1: Using Models

Model each expression using the opposite value of each tile in the polynomial you are subtracting.

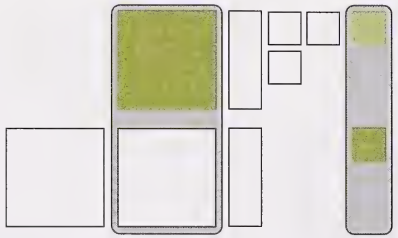
x^2-x-4

$-2x^2-x+1$



The opposite of $2x^2+x-1$ is $-2x^2-x+1$.

Combine the two sets of tiles, and simplify by removing the zero pairs.



This is the result.

$\therefore \left(x^2-x-4\right)-\left(2x^2+x-1\right)=-x^2-2x-3$

Method 2: Using Paper and Pencil

$$\begin{aligned}\left(x^2-x-4\right)-\left(2x^2+x-1\right)&=\left(x^2-x-4\right)+\left(-2x^2-x+1\right)\\&=x^2-x-4-2x^2-x+1\\&=x^2-2x^2-x-x-4+1\\&=-x^2-2x-3\end{aligned}$$

c. Method 1: Using Models

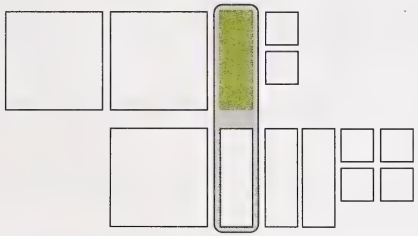
Model each expression using the opposite value of each tile in the polynomial you are subtracting.

$-2x^2+x-2$

$-x^2-3x-4$

The opposite of x^2+3x+4 is $-x^2-3x-4$.

Combine the two sets of tiles, and simplify by removing the zero pairs.



This is the result.

$\therefore \left(-2x^2+x-2\right)-\left(x^2+3x+4\right)=-3x^2-2x-6$

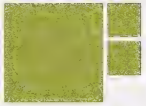
Method 2: Using Paper and Pencil

$$\begin{aligned}\left(-2x^2+x-2\right)-\left(x^2+3x+4\right)&=\left(-2x^2+x-2\right)+\left(-x^2-3x-4\right)\\&=-2x^2+x-2-x^2-3x-4\\&=-2x^2-x^2+x-3x-2-4\\&=-3x^2-2x-6\end{aligned}$$

d. Method 1: Using Models

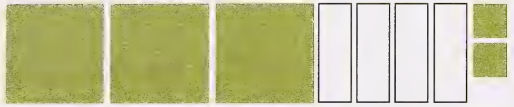
Model each expression using the opposite value of each tile in the second polynomial.

x^2+2



$2x^2-4x$

Combine the two sets of tiles, and simplify.



$\therefore \left(x^2+2\right)-\left(-2x^2+4x\right)=3x^2-4x+2$

Method 2: Using Paper and Pencil

$$\begin{aligned}\left(x^2+2\right)-\left(-2x^2+4x\right)&=\left(x^2+2\right)+\left(2x^2-4x\right)\\&=x^2+2+2x^2-4x\\&=x^2+2x^2-4x+2\\&=3x^2-4x+2\end{aligned}$$

7. a. $\left(4y^2+3y+5\right)-\left(y^2+3y+2\right)=\left(4y^2+3y+5\right)+\left(-y^2-3y-2\right)$

$$\begin{aligned}&=4y^2+3y+5-y^2-3y-2\\&=4y^2-y^2+3y-3y+5-2\\&=3y^2+3\end{aligned}$$

b. $\left(-3y^2-y+7\right)-\left(2y^2-3y+3\right)=\left(-3y^2-y+7\right)+\left(-2y^2+3y-3\right)$

$$\begin{aligned}&=-3y^2-y+7-2y^2+3y-3\\&=-3y^2-2y^2-y+3y+7-3\\&=-5y^2+2y+4\end{aligned}$$

c. $\left(3y^3+3y^2-2y+5\right)-\left(y^3-2y^2+4y-2\right)$

$$\begin{aligned}&=\left(3y^3+3y^2-2y+5\right)+\left(-y^3+2y^2-4y+2\right)\\&=3y^3+3y^2-2y+5-y^3+2y^2-4y+2\\&=3y^3-y^3+3y^2+2y^2-2y-4y+5+2\\&=2y^3+5y^2-6y+7\end{aligned}$$

d. $\left(y^2-4y+5\right)-\left(y^2+6\right)=\left(y^2-4y+5\right)+\left(-y^2-6\right)$

$$\begin{aligned}&=y^2-4y+5-y^2-6\\&=y^2-y^2-4y+5-6\\&=-4y-1\end{aligned}$$

8. a.
$$\begin{array}{r} 4a^2+3a+5 \\ -\left(2a^2+2a+3\right) \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{r} 4a^2+3a+5 \\ +\left(-2a^2-2a-3\right) \\ \hline 2a^2+\ a+2 \end{array}$$

b.
$$\begin{array}{r} 2b^2+7b-3 \\ -\left(b^2-2b-1\right) \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{r} 2b^2+7b-3 \\ +\left(-b^2+2b+1\right) \\ \hline b^2+9b-2 \end{array}$$

c.
$$\begin{array}{r} -6c^2+5c-8 \\ -\ \ \left(4c^2+7\right) \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{r} -6c^2+5c-\ 8 \\ +\left(-4c^2\ \ \ -\ 7\right) \\ \hline -10c^2+5c-15 \end{array}$$

d. $8n^3+4n^2-2n+7$

$-\left(2n^3-3n^2+n+2\right)$

$8n^3+4n^2-2n+7$

$+\left(-2n^3+3n^2-\ n-2\right)$

$6n^3+7n^2-3n+5$

9. a. Area of shaded region $=\left(4x^2+4x+1\right)-\left(x^2+2x-3\right)$

b. $\left(4x^2+4x+1\right)-\left(x^2+2x-3\right)=\left(4x^2+4x+1\right)+\left(-x^2-2x+3\right)$

$=4x^2+4x+1-x^2-2x+3$

$=4x^2-x^2+4x-2x+1+3$

$=3x^2+2x+4$

c. Area of shaded region $=3x^2+2x+4$

$=3(4)^2+2(4)+4$

$=3(16)+8+4$

$=48+8+4$

$=60\ \text{cm}^2$

10. a. Area of large circle $=\pi r^2$

$=\pi(x)^2$

$=\pi x^2$

Area of one small circle $=\pi r^2$

$=\pi\left(\frac{x}{2}\right)^2$

$=\frac{\pi x^2}{4}$

Area of two small circles $=2\left(\frac{\pi x^2}{4}\right)$

$=\frac{\pi x^2}{2}$

$\therefore$ Area of shaded region $=\pi x^2-\frac{\pi x^2}{2}$

b. $\pi x^2-\frac{\pi x^2}{2}=\frac{2\pi x^2}{2}-\frac{\pi x^2}{2}$

$=\frac{\pi x^2}{2}$

The area of the shaded region is the same as the area of the two small circles.

Now Try This

11. a. **i.** 30 **ii.** 18 **iii.** 23
iv. 14 **v.** 19 **vi.** 9
vii. 7 **viii.** 8 **ix.** 5
x. 3 **xi.** 4

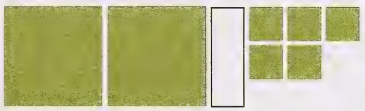
b. 1

Looking Back

12. a. The student did not change the sign of each term in the polynomial being subtracted.

b. Model the two expressions.

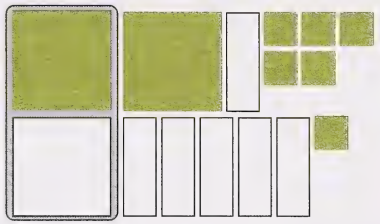
$2x^2 - x + 5$



$-x^2 - 5x + 1$

The opposite of $x^2 + 5x - 1$ is $-x^2 - 5x + 1$.

Combine the two sets of tiles, and simplify by removing the zero pairs.



This is the result.

$\therefore \left(2x^2 - x + 5\right) - \left(x^2 + 5x - 1\right) = x^2 - 6x + 6$

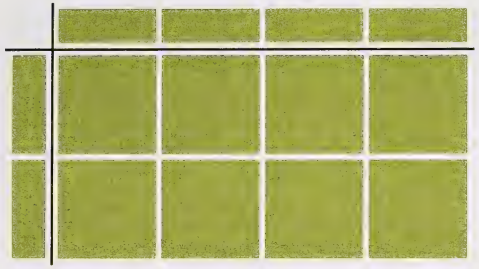
c. $\left(2x^2 - x + 5\right) - \left(x^2 + 5x - 1\right) = 2x^2 - x + 5 + \left(-x^2 - 5x + 1\right)$

$= 2x^2 - x^2 - x - 5x + 5 + 1$

$= x^2 - 6x + 6$

Section 1: Activity 4

1. a. Method 1: Using Models



$\therefore\ (2x)(4x) = 8x^2$

Method 2: Using Paper and Pencil

$$(2x)(4x) = (2)(4)(x)(x)$$
$$= 8x^2$$

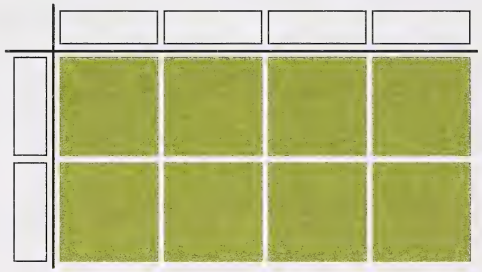
b. Method 1: Using Models

$\therefore\ (-2x)(4x) = -8x^2$

Method 2: Using Paper and Pencil

$$(-2x)(4x) = (-2)(4)(x)(x)$$
$$= -8x^2$$

c. Method 1: Using Models



$\therefore\ (-2x)(-4x) = 8x^2$

Method 2: Using Paper and Pencil

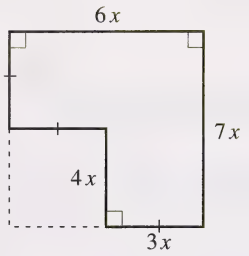
$$(-2x)(-4x) = (-2)(-4)(x)(x)$$
$$= 8x^2$$

2. **a.** $4mn$ **b.** $12a^4$ **c.** $18xy^2$ **d.** $-8x^3y^2$
e. $10a^2b^2$ **f.** $12x^2yz$ **g.** $-8ab^2$ **h.** $12x^2y^5$
i. $6a^2bc$ **j.** $12a^3b^5c^2$ **k.** $24m^4$ **l.** $-6a^6b^3$
m. $4a^2$ **n.** $8a^3$

3. a. $(2x)(3y) = 6xy$

b. $(3y)^2 = 9y^2$

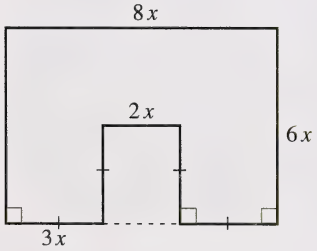
c.



$$(6x)(7x) - (3x)(4x) = 42x^2 - 12x^2$$
$$= 30x^2$$

d. $(7ab)(4) = 28ab$

e.



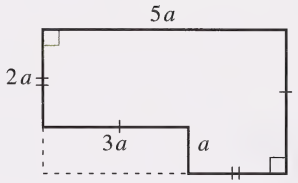
$$(8x)(6x) - (3x)(2x) = 48x^2 - 6x^2$$
$$= 42x^2$$

4. a.

$$P = 2\ell + 2w$$
$$= 2(5m) + 2(2m)$$
$$= 10m + 4m$$
$$= 14m$$

$$A = \ell \times w$$
$$= (5m)(2m)$$
$$= 10m^2$$

b.



$$P = 5a + 3a + 2a + a + 3a + 2a$$
$$= 16a$$

$$A = (5a)(3a) - (3a)(a)$$
$$= 15a^2 - 3a^2$$
$$= 12a^2$$

5. Answers will vary. Some examples are provided.

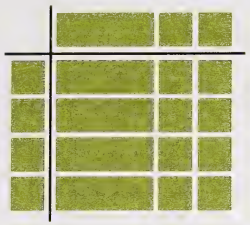
- $(3a)(2b)$
- $(2ab)(3)$
- $(a)(6b)$
- $(ab)(6)$
- $(2a)(3b)$
- $(3ab)(2)$
- $(6a)(b)$
- $(6ab)(1)$

Looking Back

6. The product you are modelling is $(3x)(2x) = 6x^2$.

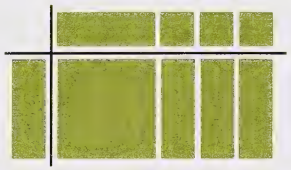
Section 1: Activity 5

1. a.



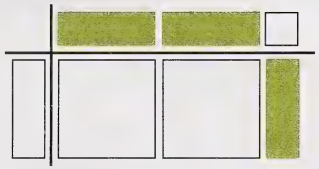
$\therefore\ 4(x+2)=4x+8$

b.



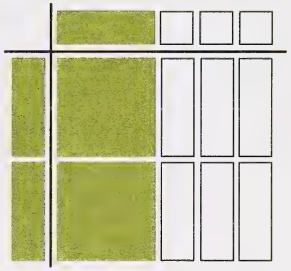
$\therefore\ x(x+3)=x^2+3x$

c.



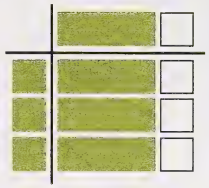
$\therefore\ -x(2x-1)=-2x^2+x$

d.



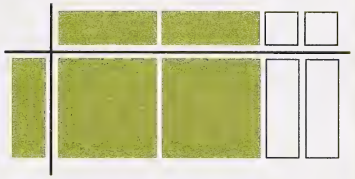
$\therefore\ 2x(x-3)=2x^2-6x$

e.



$\therefore\ 3(x-1)=3x-3$

f.



$\therefore\ x(2x-2)=2x^2-2x$

2. a. $4(x-1)=4(x-1)$
$=4x-4$

b. $a(a+4)=a(a+4)$
$=a^2+4a$

c. $5(x^2+x+1)=5(x^2+x+1)$
$=5x^2+5x+5$

d. $2b(b-3)=2b(b-3)$
$=2b^2-6b$

e. $a(2a+3)=a(2a+3)$
$=2a^2+3a$

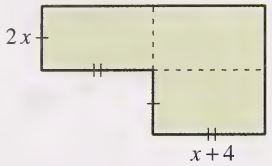
f. $2y(y^2-y+3)=2y(y^2-y+3)$
$=2y^3-2y^2+6y$

3. a. $2(a^2+2a+3)+3(a^2-a+1)=2(a^2+2a+3)+3(a^2-a+1)$
$=2a^2+4a+6+3a^2-3a+3$
$=5a^2+a+9$

b. $-3a(a^2+a-4)+2(a+1)=-3a(a^2+a-4)+2(a+1)$
$=-3a^3-3a^2+12a+2a+2$
$=-3a^3-3a^2+14a+2$

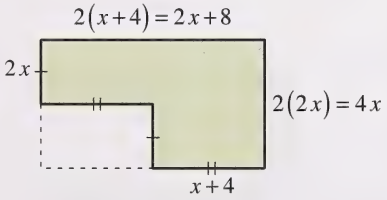
4. a. $3x(x+3)=3x^2+9x$

b. Method 1



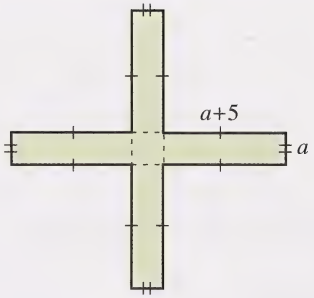
$$3\left[2x(x+4)\right]=3\left(2x^2+8x\right)$$
$$=6x^2+24x$$

Method 2



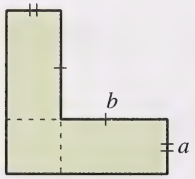
$$4x(2x+8)-2x(x+4)=8x^2+32x-2x^2-8x$$
$$=8x^2-2x^2+32x-8x$$
$$=6x^2+24x$$

c.



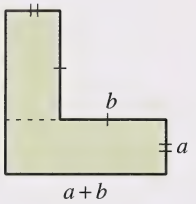
$$a(a+5)+a(a+5)+a(a+5)+a(a+5)+a(a)$$
$$=a^2+5a+a^2+5a+a^2+5a+a^2+5a+a^2$$
$$=5a^2+20a$$

5. Method 1



$$2\left[a(b)\right]+a(a)$$
$$=2ab+a^2$$
$$=a^2+2ab$$

Method 2



$$a(b)+a(a+b)$$
$$=ab+a^2+ab$$
$$=a^2+2ab$$

6. $\text{Volume}=(4x)(4x)(4x)$
$$=64x^3$$

7. a. $a(a+1)=a^2+a$

Note: If $a=4$, then $a+1=5$. These numbers are consecutive.

b. Square the smallest number; then add it to the smallest number.

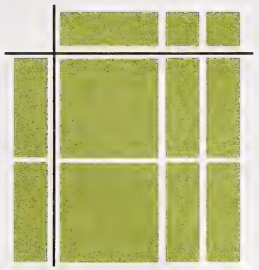
c. Try the rule with -4 and -3.

$$(-4)\times(-3)=12 \qquad (-4)^2+(-4)=16+(-4)$$
$$=12$$

Yes, the rule applies to negative integers as well.

Looking Back

8. Your diagram should look like the following.

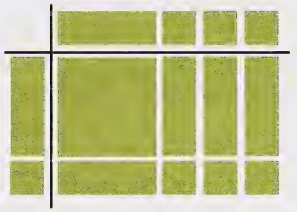


The length is $x+2$, and the width is $2x$.

$$\therefore\ 2x(x+2)=2x(x+2)$$
$$=2x^2+4x$$

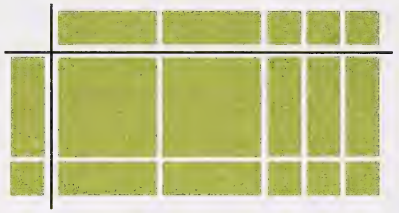
Section 1: Activity 6

1. a.



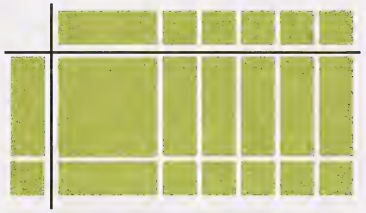
$$\therefore\ (x+3)(x+1)=x^2+4x+3$$

b.



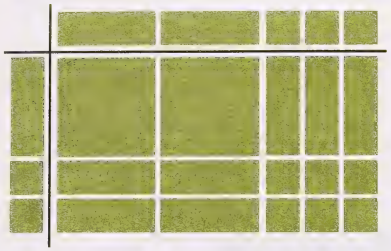
$$\therefore\ (2x+3)(x+1)=2x^2+5x+3$$

2. a.



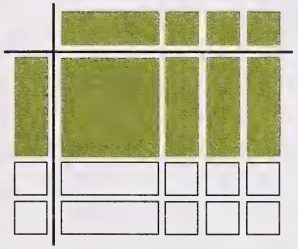
$$(x+5)(x+1)=x(x)+x(1)+5(x)+5(1)$$
$$=x^2+x+5x+5$$
$$=x^2+6x+5$$

b.



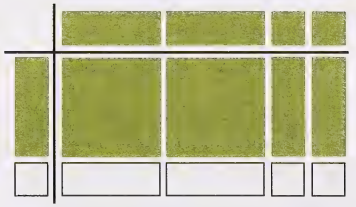
$$\begin{aligned}(2x+3)(x+2) &= 2x(x)+2x(2)+3(x)+3(2)\\ &= 2x^2+4x+3x+6\\ &= 2x^2+7x+6\end{aligned}$$

c.



$$\begin{aligned}(x+3)(x-2) &= x(x)+x(-2)+3(x)+3(-2)\\ &= x^2-2x+3x-6\\ &= x^2+x-6\end{aligned}$$

d.



$$\begin{aligned}(2x+2)(x-1) &= 2x(x)+2x(-1)+2(x)+2(-1)\\ &= 2x^2-2x+2x-2\\ &= 2x^2-2\end{aligned}$$

3. a. $(x+4)(x+3) = \underbrace{x(x)}_{\text{F}} + \underbrace{x(3)}_{\text{O}} + \underbrace{4(x)}_{\text{I}} + \underbrace{4(3)}_{\text{L}}$

$$\begin{aligned}&= x^2+3x+4x+12\\ &= x^2+7x+12\end{aligned}$$

b. $(x-2)(x+5) = \underbrace{x(x)}_{\text{F}} + \underbrace{x(5)}_{\text{O}} + \underbrace{(-2)(x)}_{\text{I}} + \underbrace{(-2)(5)}_{\text{L}}$

$$\begin{aligned}&= x^2+5x-2x-10\\ &= x^2+3x-10\end{aligned}$$

c. $(2x-3)(x+4) = \underbrace{2x(x)}_{\text{F}} + \underbrace{2x(4)}_{\text{O}} + \underbrace{(-3)(x)}_{\text{I}} + \underbrace{(-3)(4)}_{\text{L}}$

$$\begin{aligned}&= 2x^2+8x-3x-12\\ &= 2x^2+5x-12\end{aligned}$$

d. $(x+6)(x-1) = \underbrace{x(x)}_{\text{F}} + \underbrace{x(-1)}_{\text{O}} + \underbrace{6(x)}_{\text{I}} + \underbrace{6(-1)}_{\text{L}}$

$= x^2 - x + 6x - 6$

$= x^2 + 5x - 6$

e. $(2x+6)(x+2) = \underbrace{2x(x)}_{\text{F}} + \underbrace{2x(2)}_{\text{O}} + \underbrace{6(x)}_{\text{I}} + \underbrace{6(2)}_{\text{L}}$

$= 2x^2 + 4x + 6x + 12$

$= 2x^2 + 10x + 12$

f. $(x-1)(x-1) = \underbrace{x(x)}_{\text{F}} + \underbrace{x(-1)}_{\text{O}} + \underbrace{(-1)(x)}_{\text{I}} + \underbrace{(-1)(-1)}_{\text{L}}$

$= x^2 - x - x + 1$

$= x^2 - 2x + 1$

4. a. n^2 means "n times n."

b. $(x+2)(x+2) = x(x) + x(2) + 2(x) + 2(2)$

$= x^2 + 2x + 2x + 4$

$= x^2 + 4x + 4$

c. $(x+3)(x+3) = x(x) + x(3) + 3(x) + 3(3)$

$= x^2 + 3x + 3x + 9$

$= x^2 + 6x + 9$

d. $(x+4)^2 = (x+4)(x+4)$

$= x(x) + x(4) + 4(x) + 4(4)$

$= x^2 + 4x + 4x + 16$

$= x^2 + 8x + 16$

Therefore, $x^2 + 8x + 16$ is a perfect square trinomial.

5. a. $(x-4)(x+4) = x^2 + 4x - 4x - 16$

$= x^2 - 16$

b. $(a+3)(a-3) = a^2 - 3a + 3a - 9$

$= a^2 - 9$

c. $(2x-1)(2x+1) = 4x^2 + 2x - 2x - 1$

$= 4x^2 - 1$

d. $(y-2)(y+2) = y^2 + 2y - 2y - 4$

$= y^2 - 4$

6. a. The similarities are as follows:

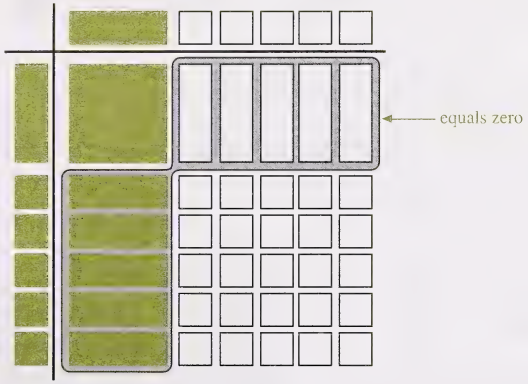
- There are only two terms in each answer.
- The product is always a difference of those two terms.

b. They are both perfect squares.

c. Yes, $(x-5)(x+5)=x^2-25$

Both terms are perfect squares.

d.



The equal numbers of x-tiles and $-x$-tiles add up to zero.

Therefore, you are left with x^2-25.

7. a. $(2x+3)(2x-3)=2x(2x)+2x(-3)+3(2x)+(3)(-3)$

$=4x^2-6x+6x-9$

$=4x^2-9$

b. $(x+4)(x-1)=x(x)+x(-1)+4(x)+4(-1)$

$=x^2-x+4x-4$

$=x^2+3x-4$

c. $(x+3)(x-5)=x(x)+x(-5)+(3)(x)+(3)(-5)$

$=x^2-5x+3x-15$

$=x^2-2x-15$

d. $(x+7)^2=(x+7)(x+7)$

$=x(x)+x(7)+7(x)+7(7)$

$=x^2+7x+7x+49$

$=x^2+14x+49$

e. $(2x+5)(x+2)=2x(x)+2x(2)+5(x)+5(2)$

$=2x^2+4x+5x+10$

$=2x^2+9x+10$

f. $(x-9)(x-6)=x(x)+x(-6)+(-9)(x)+(-9)(-6)$

$=x^2-6x-9x+54$

$=x^2-15x+54$

8. a. $P = 2(x+2)+2(x+6)$
$= 2x+4+2x+12$
$= 4x+16$

$A = (x+2)(x+6)$
$= x^2+6x+2x+12$
$= x^2+8x+12$

b. $P = 4(x+3)$
$= 4x+12$

$A = (x+3)^2$
$= (x+3)(x+3)$
$= x^2+3x+3x+9$
$= x^2+6x+9$

Now Try This

9. Method 1: Using Algebra

Let n be the smaller number, and let $n+1$ represent the larger number.

$$(n+1)^2 - n^2 = 81$$
$$n^2+2n+1-n^2 = 81$$
$$2n+1 = 81$$
$$2n = 80$$
$$n = 40$$

The numbers are 40 and 41.

Method 2: Guessing, Checking, and Revising

$30^2 - 29^2 = 900 - 841 = 59$ (too low)

$50^2 - 49^2 = 2500 - 2401 = 99$ (too high)

$40^2 - 39^2 = 1600 - 1521 = 79$ (too low, but close)

$41^2 - 40^2 = 1681 - 1600 = 81$

Method 3: Using Patterns

1 4 9 16 25 36 49

The difference between the first and second perfect squares is 3.
The difference between the second and third perfect squares is 5.
The difference between the third and fourth perfect squares is 7.

$\therefore\ 1+2=3$
$2+3=5$
$3+4=7$

Thus, the difference between the 40th and 41st perfect squares would be 81.

10. Make an organized list of the possible combinations of Marten's placements.

First	Second	Third
3	0	0
2	1	1
1	2	2
1	0	5
0	5	0
0	3	3
0	1	6

There are seven possible combinations that result in Marten receiving a total of 15 points.

Looking Back

11. a. $(2x-1)(x+5) = \underbrace{2x(x)}_{\text{F}} + \underbrace{2x(5)}_{\text{O}} + \underbrace{(-1)(x)}_{\text{I}} + \underbrace{(-1)(5)}_{\text{L}}$

$= 2x^2 + 10x - x - 5$

$= 2x^2 + 9x - 5$

b. $(x+5)^2 = (x+5)(x+5)$

$= \underbrace{x(x)}_{\text{F}} + \underbrace{x(5)}_{\text{O}} + \underbrace{5(x)}_{\text{I}} + \underbrace{5(5)}_{\text{L}}$

$= x^2 + 5x + 5x + 25$

$= x^2 + 10x + 25$

This is an example of a perfect square. To avoid common errors, it is important to write the expression as a product of two factors first.

c. $(x-6)(x+6) = \underbrace{x(x)}_{\text{F}} + \underbrace{x(6)}_{\text{O}} + \underbrace{(-6)(x)}_{\text{I}} + \underbrace{(-6)(6)}_{\text{L}}$

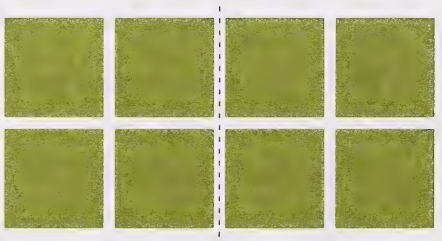
$= x^2 + 6x - 6x - 36$

$= x^2 - 36$

A difference of squares has two terms because the sum of the outer and inner terms is zero.

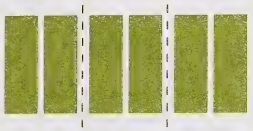
Section 1: Activity 7

1. a.



$\therefore\ 8x^2 \div 2 = 4x^2$

b.



$\therefore\ 6x \div 3 = 2x$

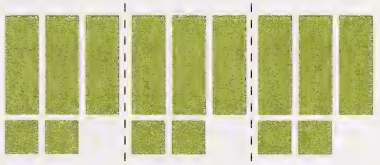
c.

$\therefore\ \left(2x^2 + 4\right) \div 2 = x^2 + 2$

d.

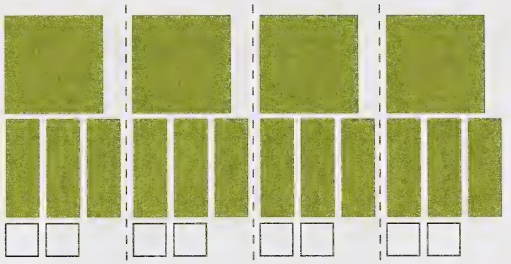
$\therefore\ \left(-2x^2 - 6x + 4\right) \div 2 = -x^2 - 3x + 2$

e.



$\therefore\ \left(9x + 6\right) \div 3 = 3x + 2$

f.



$\therefore\ \left(4x^2 + 12x - 8\right) \div 4 = x^2 + 3x - 2$

2. Divide the numerical coefficient of each term by the constant.

3. a. $\left(6x - 9\right) \div 3 = \dfrac{6x}{3} + \dfrac{-9}{3}$

$= 2x - 3$

b. $\left(4x^2 + 2x - 2\right) \div 2 = \dfrac{4x^2}{2} + \dfrac{2x}{2} + \dfrac{-2}{2}$

$= 2x^2 + x - 1$

c. $\dfrac{6x^2 + 9x - 3}{3} = \dfrac{6x^2}{3} + \dfrac{9x}{3} + \dfrac{-3}{3}$

$= 2x^2 + 3x - 1$

4. a. $(12y-8)\div 4=\dfrac{12y}{4}+\dfrac{-8}{4}$

$=3y-2$

The length of a side is $3y-2$ units.

b. $A=(3y-2)(3y-2)$

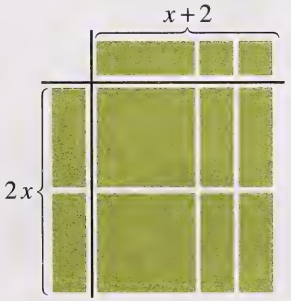
$=3y(3y)+3y(-2)+(-2)(3y)+(-2)(-2)$

$=9y^2-6y-6y+4$

$=9y^2-12y+4$

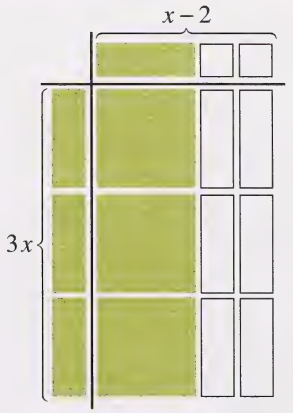
The area of the square is $9y^2-12+4$ square units.

5. a.



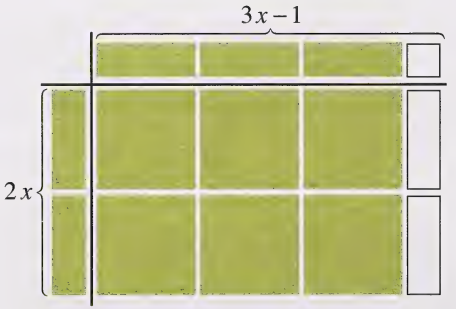
$\therefore\ (2x^2+4x)\div 2x=x+2$

b.



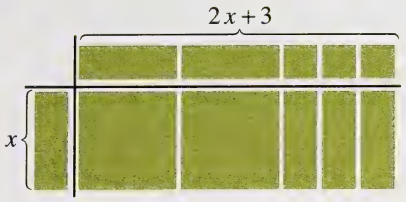
$\therefore\ (3x^2-6x)\div 3x=x-2$

c.



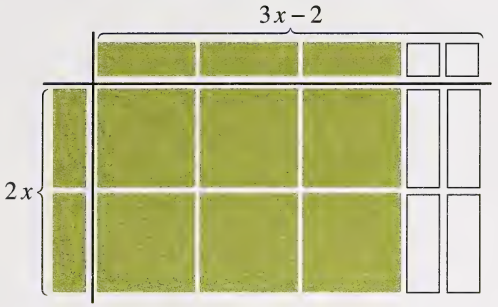
$\therefore\ (6x^2-2x)\div 2x=3x-1$

d.



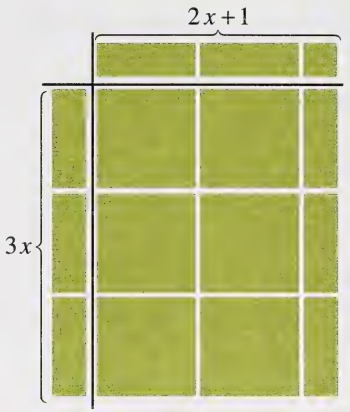
$\therefore \left(2x^2 + 3x\right) \div x = 2x + 3$

6. a.



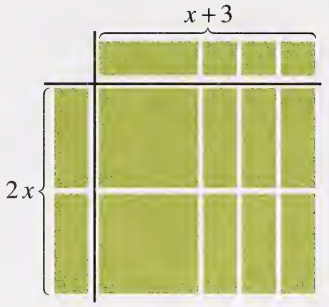
$\therefore \left(6x^2 - 4x\right) \div 2x = 3x - 2$

b.



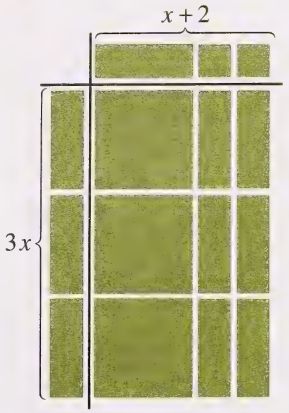
$\therefore \left(6x^2 + 3x\right) \div 3x = 2x + 1$

c.



$\therefore \left(2x^2 + 6x\right) \div 2x = x + 3$

d.



$\therefore \left(3x^2+6x\right)\div 3x = x+2$

7. a. $\left(6x^2+5x\right)\div x = \dfrac{6x^2}{x}+\dfrac{5x}{x}$

$= 6x+5$

b. $\left(4x^2+6x\right)\div 2x = \dfrac{4x^2}{2x}+\dfrac{6x}{2x}$

$= 2x+3$

c. $\left(10x^2+5x\right)\div 5x = \dfrac{10x^2}{5x}+\dfrac{5x}{5x}$

$= 2x+1$

d. $\dfrac{4x^2+2x+6}{2} = \dfrac{4x^2}{2}+\dfrac{2x}{2}+\dfrac{6}{2}$

$= 2x^2+x+3$

e. $\dfrac{3x^2-6x}{3x} = \dfrac{3x^2}{3x}+\dfrac{-6x}{3x}$

$= x-2$

f. $\dfrac{5x^3+10x^2-15x}{5x} = \dfrac{5x^3}{5x}+\dfrac{10x^2}{5x}+\dfrac{-15x}{5x}$

$= x^2+2x-3$

8. a. $\dfrac{4x^2+6x-8}{5} = \dfrac{4x^2}{5}+\dfrac{6x}{5}+\dfrac{-8}{5}$

$= \dfrac{4}{5}x^2+\dfrac{6}{5}x-\dfrac{8}{5}$

Multiply to verify your answer.

$5\left(\dfrac{4}{5}x^2+\dfrac{6}{5}x-\dfrac{8}{5}\right) = 5\left(\dfrac{4}{5}x^2\right)+5\left(\dfrac{6}{5}x\right)+5\left(-\dfrac{8}{5}\right)$

$= 4x^2+6x-8$

b. $\dfrac{8x^3+12x^2-4x}{-4x} = \dfrac{8x^3}{-4x}+\dfrac{10x^2}{-4x}+\dfrac{-4x}{-4x}$

$= -2x^2-\dfrac{5}{2}x+1$

Multiply to verify your answer.

$$-4x\left(-2x^2-3x+1\right)=(-4x)\left(-2x^2\right)+\left(\overset{-2}{\cancel{-4}}x\right)\left(-\frac{5}{\cancel{2}_{1}}x\right)+(-4x)(1)$$

$$=8x^3+10x^2-4x$$

9. a. $\left(6x^3+4x^2+8x\right)\div 3x=\dfrac{6x^3}{3x}+\dfrac{4x^2}{3x}+\dfrac{8x}{3x}$

$$=2x^2+\frac{4}{3}x+\frac{8}{3}$$

b. $\left(12x^4+8x^3-16x^2+4x\right)\div(-4x)$

$$=\frac{12x^4}{-4x}+\frac{8x^3}{-4x}+\frac{-16x^2}{-4x}+\frac{4x}{-4x}$$

$$=-3x^3-2x^2+4x-1$$

c. $\left(x^3-9x\right)\div x=\dfrac{x^3}{x}+\dfrac{-9x}{x}$

$$=x^2-9$$

d. $\dfrac{-12y^3+9y^2-y}{-3y}=\dfrac{-12y^3}{-3y}+\dfrac{9y^2}{-3y}+\dfrac{-y}{-3y}$

$$=4y^2-3y+\frac{1}{3}$$

e. $\dfrac{24-8m+4m^2}{4}=\dfrac{24}{4}+\dfrac{-8m}{4}+\dfrac{4m^2}{4}$

$$=6-2m+m^2$$

f. $\dfrac{14a^2b^3+7a^2b^2-7ab^2}{7ab}=\dfrac{14a^2b^3}{7ab}+\dfrac{7a^2b^2}{7ab}+\dfrac{-7ab^2}{7ab}$

$$=2ab^2+ab-b$$

10. a. Width $=\dfrac{3a^2+6a}{3a}$

$$=\frac{3a^2}{3a}+\frac{6a}{3a}$$

$$=a+2$$

b. Width $=\dfrac{4c^2-8c}{4c}$

$$=\frac{4c^2}{4c}\frac{-8c}{4c}$$

$$=c-2$$

11. a. Length $=\dfrac{6b^2+2b}{2b}$

$$=\frac{6b^2}{2b}+\frac{2b}{2b}$$

$$=3b+1$$

b. Perimeter $=2(2b)+2(3b+1)$

$$=4b+6b+2$$

$$=10b+2$$

12. a. $\dfrac{6x^2y^2}{2xy}=\dfrac{6(2)^2(-3)^2}{2(2)(-3)}$

$$=\frac{6(4)(9)}{2(2)(-3)}$$

$$=\frac{216}{-12}$$

$$=-18$$

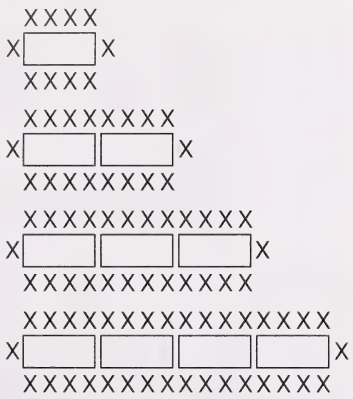
b. $3xy=3(2)(-3)$

$$=-18$$

13. The answers in question 12 are the same because $6x^2y^2\div 2xy=3xy$.

Now Try This

14. Find the pattern using one, two, three, and four tables. You may wish to draw diagrams.



Pattern
$=8+2$
$=8(2)+2$
$=8(3)+2$
$=8(4)+2$

From the pattern, the number of people that can be seated can be represented by $8n+2$, where n is the number of tables.

Use the equation to find the number of people that can be seated at 13 tables.

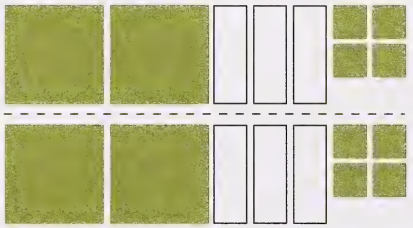
$$\begin{aligned} 8n+2 &= 8(13)+2 \\ &= 104+2 \\ &= 106 \end{aligned}$$

Therefore, a maximum of 106 people can be seated at 13 tables.

Looking Back

15. a. The polynomial is $4x^2-6x+8$.

b. The polynomial is divisible by 2 because the tiles can be divided into two groups.



The polynomial is not divisible by 4 because the six $-x$-tiles cannot be divided into groups of 4 with the same number in each group.

c. The quotient is $2x^2-3x+4$.

Section 2: Activity 1

1. a.

$$\begin{aligned} 15 &= 3\times 5 \\ 20 &= 2\times 2\times 5 \\ 35 &= 5\times 7 \end{aligned}$$

$\therefore$ GCF $= 5$

b. $12 = 2 \times 2 \times 3$

$-16 = -1 \times 2 \times 2 \times 2$

$20 = 2 \times 2 \times 5$

$\therefore \text{GCF} = 2 \times 2$

$= 4$

c. $6x = 2 \times 3 \times x$

$8x^2 = 2 \times 2 \times 2 \times x \times x$

$4x^3 = 2 \times 2 \times x \times x \times x$

$\therefore \text{GCF} = 2 \times x$

$= 2x$

d. $-9x = -1 \times 3 \times 3 \times x$

$15x^2 = 3 \times 5 \times x \times x$

$-12x^3 = -1 \times 2 \times 2 \times 3 \times x \times x \times x$

$\therefore \text{GCF} = 3 \times x$

$= 3x$

e. $24z^4 = 2 \times 2 \times 2 \times 3 \times z \times z \times z \times z$

$30z^3 = 2 \times 3 \times 5 \times z \times z \times z$

$-12z^2 = -1 \times 2 \times 2 \times 3 \times z \times z$

$\therefore \text{GCF} = 2 \times 3 \times z \times z$

$= 6z^2$

f. $9ab^2c = 3 \times 3 \times a \times b \times b \times c$

$12a^2b = 2 \times 2 \times 3 \times a \times a \times b$

$21b^2c = 3 \times 7 \times b \times b \times c$

$\therefore \text{GCF} = 3 \times b$

$= 3b$

2. a. $4a^2 + 6a = 2a(\mathbf{2a + 3})$

b. $7a^3 - 14a^2 + 28a = 7a(\mathbf{a^2 - 2a + 4})$

c. $18b + 3b^2 = \mathbf{3b}(6 + b)$

d. $12b^2 + 20b^3 - 24b^4 = \mathbf{4b^2}(3 + 5b - 6b^2)$

e. $10b^3 + 8b^2 - 12b + 6 = 2(\mathbf{5b^3 + 4b^2 - 6b + 3})$

3. a. $2x^2+8x=2x(x+4)$

Check

$$2x(x+4)=(2x)(x)+(2x)(4)$$
$$=2x^2+8x$$

b. $4ab^2-7a^2b=ab(4b-7a)$

Check

$$ab(4b-7a)=(ab)(4b)+(ab)(-7a)$$
$$=4ab^2-7a^2b$$

c. $4a^2+6a+8=2\left(2a^2+3a+4\right)$

Check

$$2\left(2a^2+3a+4\right)=(2)\left(2a^2\right)+(2)(3a)+(2)(4)$$
$$=4a^2+6a+8$$

d. $7y^2+14y^3=7y^2(1+2y)$

Check

$$7y^2(1+2y)=\left(7y^2\right)(1)+\left(7y^2\right)(2y)$$
$$=7y^2+14y^3$$

e. $4a-16=4(a-4)$

Check

$$4(a-4)=(4)(a)+(4)(-4)$$
$$=4a-16$$

f. $2xy+6x^2y^2-8x^3y^3=2xy\left(1+3xy-4x^2y^2\right)$

Check

$$2xy\left(1+3xy-4x^2y^2\right)$$
$$=(2xy)(1)+(2xy)(3xy)+(2xy)\left(-4x^2y^2\right)$$
$$=2xy+6x^2y^2-8x^3y^3$$

g. $ab+ac=a(b+c)$

Check

$$a(b+c)=(a)(b)+(a)(c)$$
$$=ab+ac$$

h. $ab+ac-2ad+4ae=a(b+c-2d+4e)$

Check

$$a(b+c-2d+4e)=(a)(b)+(a)(c)+(a)(-2d)+(a)(4e)$$
$$=ab+ac-2ad+4ae$$

4. Factor the areas of the two pens to find a common factor.

$$8x^2+6x=2x(4x+3) \text{ and } 2x^2+10x=2x(x+5)$$

There is a common factor of $2x$. Therefore, the length of the common side is $2x$.

5. $P=2\ell+2w$
$=2(\ell+w)$

6. **a.** $m+n$ **b.** $a-3$ **c.** $2x+3$

d. $d-1$ **e.** $a+3$ **f.** $x-1$

7. **a.** $(x-7)(x+3)$ **b.** $(y+3)(2y-1)$

c. $(a+7)(a+3)$ **d.** $(2n+3)(n+3)$

e. $(m+1)(m-3)$ **f.** $(t+3)(2t+5)$

Now Try This

8. **a.** 16 **b.** $8^2=64$

c. $20^2=400$ **d.** $2n-1$

e. The sum is equal to n^2.

Looking Back

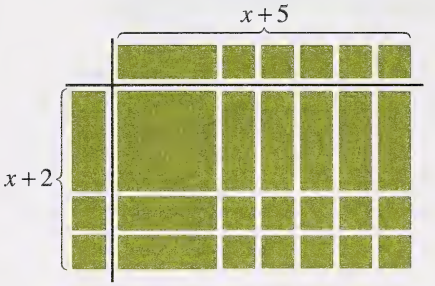
9. Answers may vary. Sample answers are given.

a. $9a^2+3a=3a(3a+1)$

b. $2a+5$

Section 2: Activity 2

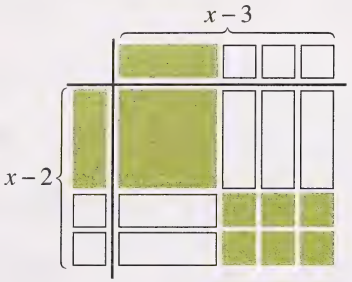
1. **a.** The trinomial represented is $x^2+7x+10$.



The length and width of the rectangular model are $x+5$ and $x+2$ respectively.

Therefore, $x^2+7x+10=(x+5)(x+2)$.

b. The trinomial represented is $x^2 - 5x + 6$.

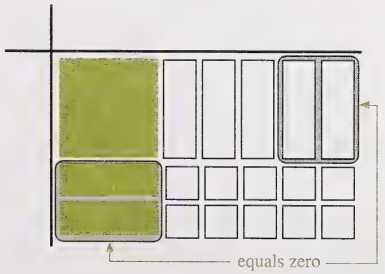


The length and width of the rectangular model are $x-3$ and $x-2$ respectively.

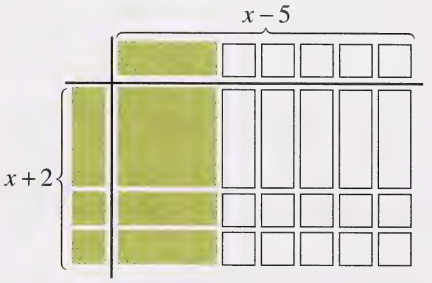
Therefore, $x^2 - 5x + 6 = (x-3)(x-2)$.

c. The trinomial represented is $x^2 - 3x - 10$.

The completed rectangle is as follows:



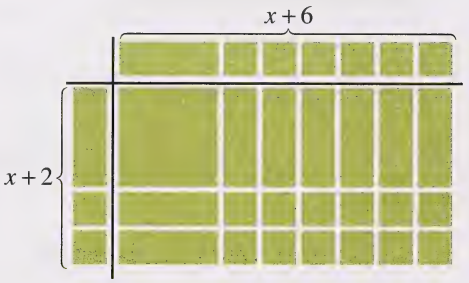
Draw in the length and width as follows:



The length and width of the rectangular model are $x-5$ and $x+2$ respectively.

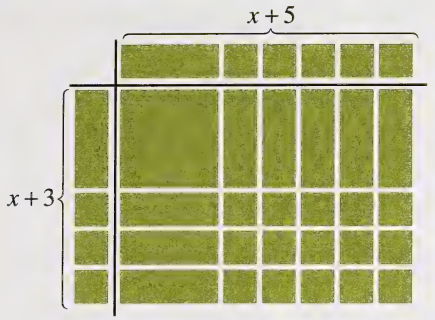
Therefore, $x^2 - 3x - 10 = (x-5)(x+2)$.

2. a.



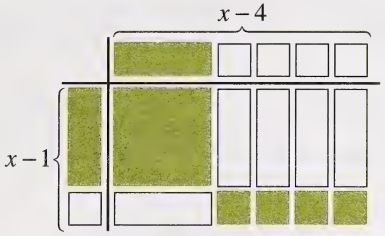
$\therefore\ x^2 + 8x + 12 = (x+6)(x+2)$

b.



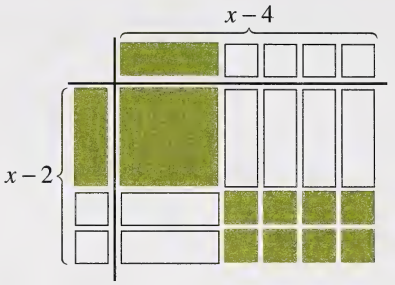
$\therefore\ x^2 + 8x + 15 = (x+5)(x+3)$

c.



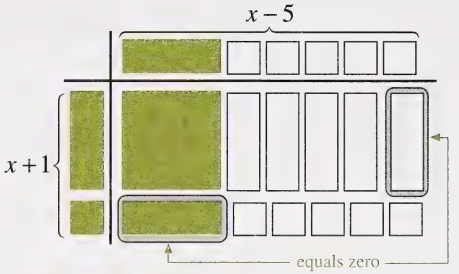
$\therefore\ x^2 - 5x + 4 = (x-4)(x-1)$

d.



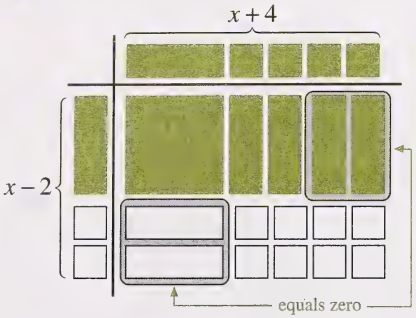
$\therefore\ x^2 - 6x + 8 = (x-4)(x-2)$

e.



$\therefore\ x^2 - 4x - 5 = (x-5)(x+1)$

f.



$\therefore\ x^2 + 2x - 8 = (x+4)(x-2)$

3. Annette's arrangement is correct because her tiles form a rectangle. When using algebra tiles for factoring, a complete rectangle or square must be formed.

4. When the constant terms of both binomial factors are added together, you get the coefficient of the middle term of the trinomial, if the trinomial is written in descending order. When the constant terms of both binomial factor are multiplied, you get the last term of the trinomial.

5. a. The factors of 16 are 16 and 1, 8 and 2, and 4 and 4.

b. The factors that add up to 10 are 8 and 2.

6. a. $x^2 + 11x + 30 = (x + 5)(x + 6)$

b. $x^2 - 9x + 20 = (x - 5)(x - 4)$

c. $n^2 - 3n - 18 = (n - 6)(n + 3)$

d. $y^2 + 5y - 14 = (y + 7)(y - 2)$

7. a. $x^2 + 13x + 36 = (x+4)(\mathbf{x+9})$

b. $x^2 - 14x - 32 = (x-16)(\mathbf{x+2})$

c. $y^2 - 11y + 24 = (y-3)(\mathbf{y-8})$

8. a. $x^2 - 12x + 20 = (x-10)(x-2)$

Check

$$(x-10)(x-2) = x^2 - 2x - 10x + 20$$
$$= x^2 - 12x + 20$$

b. $n^2 - 4n - 12 = (n-6)(n+2)$

Check

$$(n-6)(n+2) = n^2 + 2n - 6n - 12$$
$$= n^2 - 4n - 12$$

c. $y^2 + 11y - 12 = (y+12)(y-1)$

Check

$$(y+12)(y-1) = y^2 - y + 12y - 12$$
$$= y^2 + 11y - 12$$

d. $m^2 - 6m + 9 = (m-3)(m-3)$

$= (m-3)^2$

Check

$(m-3)(m-3) = m^2 - 3m - 3m + 9$

$= m^2 - 6m + 9$

e. $p^2 + 8p + 16 = (p+4)(p+4)$

$= (p+4)^2$

Check

$(p+4)(p+4) = p^2 + 4p + 4p + 16$

$= p^2 + 8p + 16$

f. $b^2 + b - 56 = (b+8)(b-7)$

Check

$(b+8)(b-7) = b^2 - 7b + 8b - 56$

$= b^2 + b - 56$

9. a. $3x^2 + 15x + 12 = 3(x^2 + 5x + 4)$

$= 3(x+4)(x+1)$

b. $2x^2 - 20x + 50 = 2(x^2 - 10x + 25)$

$= 2(x-5)(x-5)$

$= 2(x-5)^2$

c. $2x^3 + 14x^2 + 12x = 2x(x^2 + 7x + 6)$

$= 2x(x+6)(x+1)$

d. $3x^3 + 21x^2 - 54x = 3x(x^2 + 7x - 18)$

$= 3x(x+9)(x-2)$

e. $6y^2 + 12y + 6 = 6(y^2 + 2y + 1)$

$= 6(y+1)(y+1)$

$= 6(y+1)^2$

f. $x^2y + 3xy - 40y = y(x^2 + 3x - 40)$

$= y(x+8)(x-5)$

g. $4x^2 - 28x + 48 = 4(x^2 - 7x + 12)$

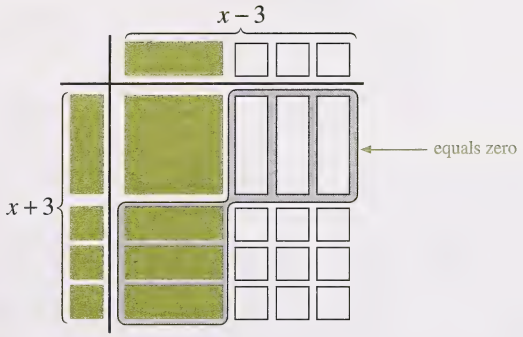
$= 4(x-3)(x-4)$

h. $x^3 + 2x^2 + x = x(x^2 + 2x + 1)$

$= x(x+1)(x+1)$

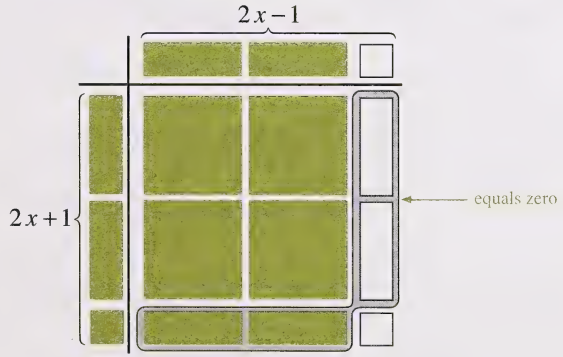
$= x(x+1)^2$

10. a.



$\therefore\ x^2 - 9 = (x+3)(x-3)$

b.



$\therefore\ 4x^2 - 1 = (2x+1)(2x-1)$

11. a. $x^2 - 49 = (x - 7)(x + 7)$

b. $y^2 - 100 = (y - 10)(y + 10)$

12. a. $4n^2 - 25 = (2n+5)(\mathbf{2n-5})$

b. $x^2 - 64 = (x-8)(\mathbf{x+8})$

c. $64y^2 - z^2 = (\mathbf{8y-z})(8y+z)$

13. a. $4x^2 - 9 = (2x-3)(2x+3)$

Check

$(2x-3)(2x+3) = 4x^2 + 6x - 6x - 9$

$= 4x^2 - 9$

b. $n^2 - 1 = (n-1)(n+1)$

Check

$(n-1)(n+1) = n^2 + n - n - 1$

$= n^2 - 1$

c. $4a^2 - b^2 = (2a - b)(2a + b)$

Check

$$(2a - b)(2a + b) = 4a^2 + 2ab - 2ab - b^2$$
$$= 4a^2 - b^2$$

d. $x^2 - y^2 = (x - y)(x + y)$

Check

$$(x - y)(x + y) = x^2 + xy - xy - y^2$$
$$= x^2 - y^2$$

14. a. $x^3 - 4x = x(x^2 - 4)$
$$= x(x - 2)(x + 2)$$

b. $6 - 24x^2 = 6(1 - 4x^2)$
$$= 6(1 - 2x)(1 + 2x)$$

c. $3x^2 - 3y^2 = 3(x^2 - y^2)$
$$= 3(x - y)(x + y)$$

d. $5n^2 - 500 = 5(n^2 - 100)$
$$= 5(n - 10)(n + 10)$$

Now Try This

15. a. Draw the tiles for $x^2 + 4x - 5$.

Add one x-tile and one $-x$-tile.

b. The dimensions are $x + 5$ and $x - 1$.

c. Complete the rectangle by adding as many zero pairs of positive and negative x-tiles or 1-tiles as are necessary.

Looking Back

16. You need two numbers whose product is 12 and whose sum is -7. These numbers are -3 and -4.

$\therefore\ x^2 - 7x + 12 = (x-3)(x-4)$

Check using FOIL.

$$\begin{aligned}(x-3)(x-4) &= x(x) + x(-4) + (-3)(x) + (-3)(-4)\\ &= x^2 - 4x - 3x + 12\\ &= x^2 - 7x + 12\end{aligned}$$

Credits

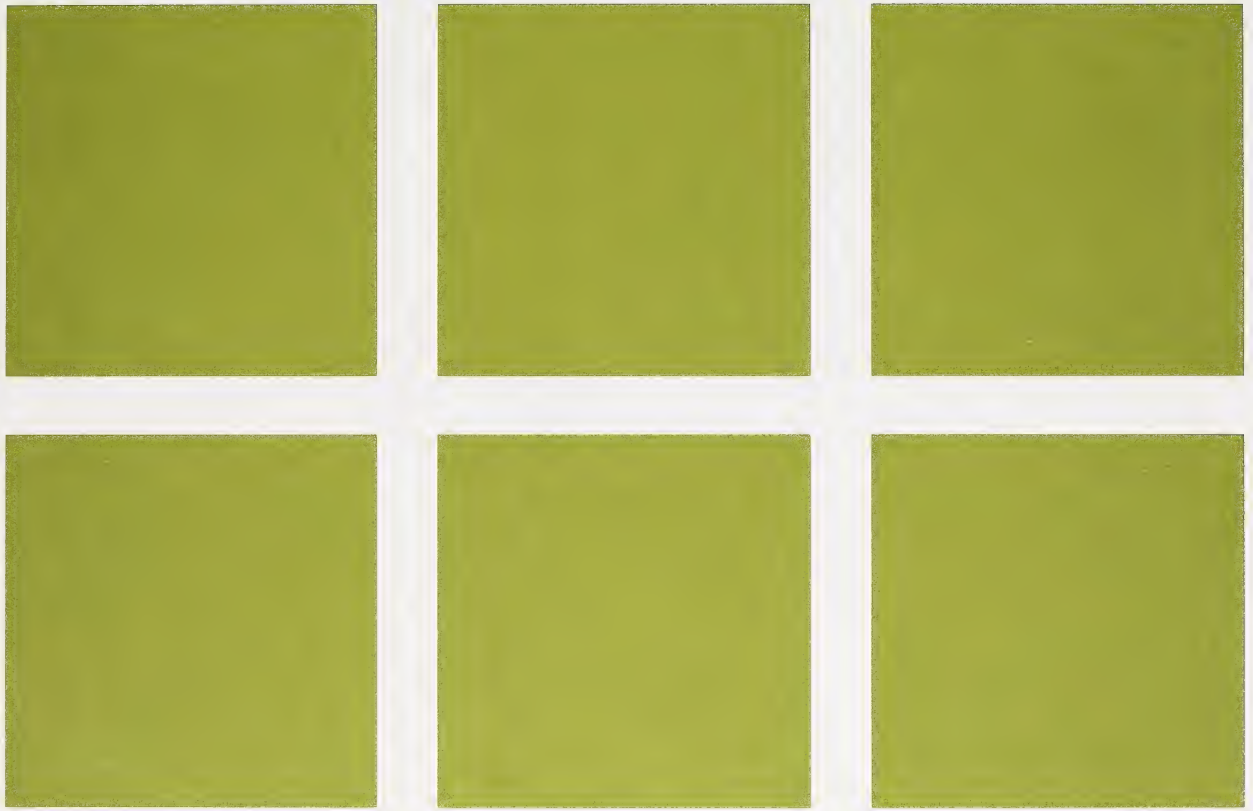
Some clip art graphics are commercially owned.

Welcome Page
PhotoDisc, Inc.

Page

1	PhotoDisc, Inc.
3	EyeWire, Inc.
5	**upper right:** PhotoDisc, Inc. **lower right:** PhotoDisc, Inc.
6	**upper left:** PhotoDisc, Inc. **lower left:** PhotoDisc, Inc.
7	**left:** PhotoDisc, Inc.
9	PhotoDisc, Inc.
10	Nova Development Corporation
11	Nova Development Corporation
12	EyeWire, Inc.
13	PhotoDisc, Inc.
15	**upper left:** PhotoDisc, Inc. **lower left:** PhotoDisc, Inc. **upper right:** PhotoDisc, Inc.
21	PhotoDisc, Inc.
22	PhotoDisc, Inc.
25	Nova Development Corporation
27	PhotoDisc, Inc.
30	**right:** PhotoDisc, Inc.
31	**right:** PhotoDisc, Inc.
35	EyeWire, Inc.
36	PhotoDisc, Inc.
41	PhotoDisc, Inc.
47	PhotoDisc, Inc.
48	PhotoDisc, Inc.
50	EyeWire, Inc.
51	Corel Corporation
52	EyeWire, Inc.
63	PhotoDisc, Inc.
64	**left:** PhotoDisc, Inc. **right:** PhotoDisc, Inc.
66	PhotoDisc, Inc.
67	PhotoDisc, Inc.
68	**left:** PhotoDisc, Inc. **right:** EyeWire, Inc.
69	Corel Corporation
71	PhotoDisc, Inc.
80	PhotoDisc, Inc.
81	**left:** PhotoDisc, Inc.
82	PhotoDisc, Inc.

x^2-Tiles

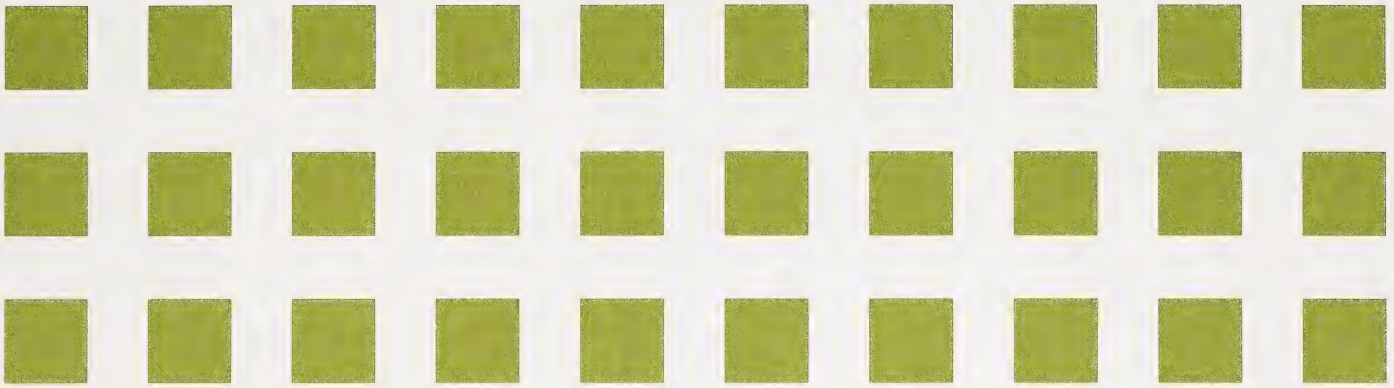


– x^2-Tiles

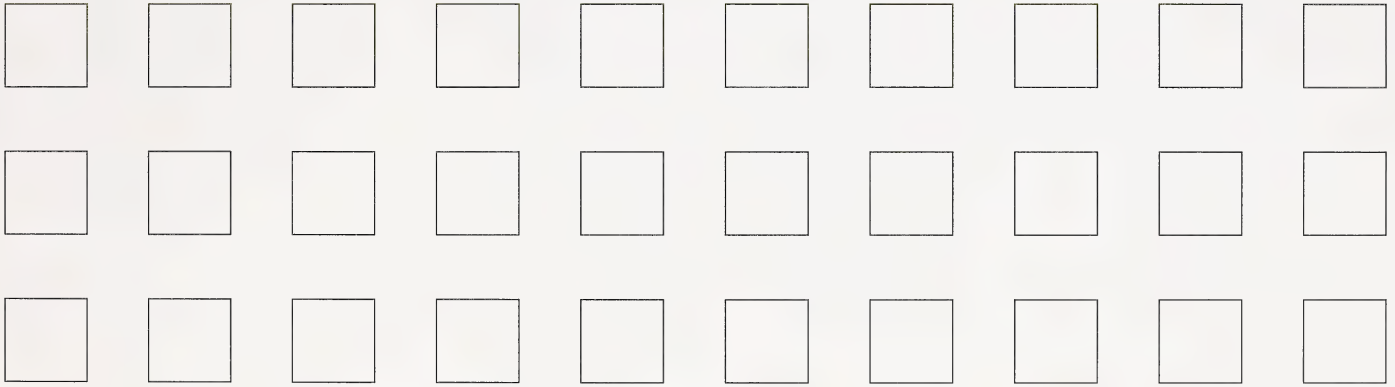
x-Tiles

– *x* -Tiles

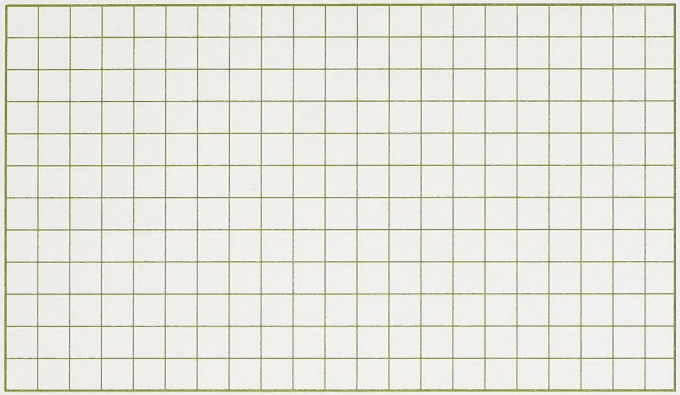
1-Tiles

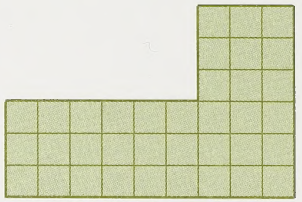


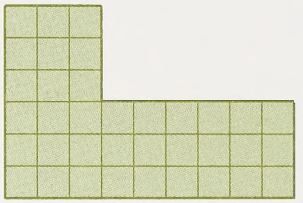
–1-Tiles

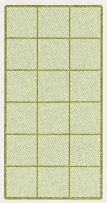


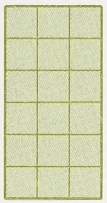
Grids and Pieces for Section 2: Activiy 2

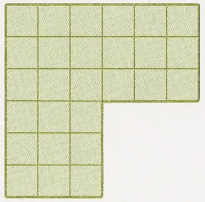


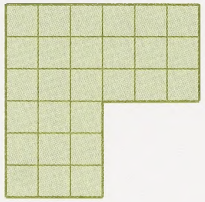


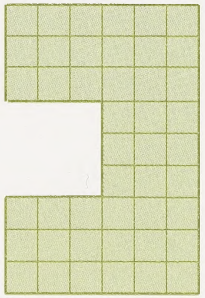


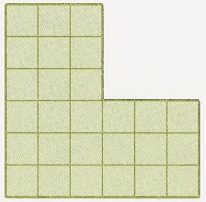












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